Tableaux for multi-agent deliberative-stit logic

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ABSTRACT. We present a sound and complete tableau calculus for the multi-agent logic of *deliberatively seeing to it that* (dstit), **Ldm**. The agents in this setting are assumed to be independent of each other. Until now only an axiomatic proof system for **Ldm** has been available. Moreover, in order to underline the usefulness of stit logics, we suggest the introduction of dstit modalities into **B**(elief)**D**(esire)**I**(ntention) logics or the introduction of belief, desire, and intention modalities into deliberative-stit logic.

Keywords: modal logic of agency, deliberative-stit logic, independence of agents, branching time structures, tableaux, **BDI** logics, belief, desire, intention.

1 Introduction

The modal logic of action reports as developed by Belnap, Perloff, and Xu, Chellas, von Kutschera, Horty and others (see [3] and references therein), has received a great deal of attention and has been applied in various areas, including deontic logic (see, for example, [1, 2, 3, 5, 8, 9, 10, 13]) and epistemic logic [6, 14, 15, 16, 17]. In [3, Chapter 17] an axiom system, **Ldm**, is presented of the logic of deliberatively seeing to it that for multiple agents satisfying the independence of agents condition on $BT+AC$ structures, i.e., branching temporal structures with agents and choice, see also [18]. In the present paper, we shall define a sound and complete tableau calculus for **Ldm**. Moreover, in order to emphasize the usefulness of dstit logic, we shall briefly motivate and discuss the introduction of dstit operators into **B**(elief)**D**(esire)**I**(ntention) logics and the introduction of belief, desire, and intention modalities into dstit logic.

2 Semantics of Ldm

We here recall the language of **Ldm** and the central semantical notions of $BT+AC$ theories and dstit logic. A more comprehensive account including motivation of the definitions may be found in [3]. The language $\mathcal{L}$ of **Ldm** is defined in Backus–Naur form as follows:

$$\begin{array}{rl} \text{atomic formulas:} & p \in Atom \\ \text{agent terms:} & \alpha \in Term \\ \text{formulas:} & A \in Form(Atom, Term) \\ A ::= & p \mid \neg A \mid (A \wedge A) \mid Sett\colon A \mid (\alpha = \alpha) \mid [\alpha\ dstit\colon A] \end{array}$$

We assume that $Term$ is a finite set. $Sett$: ("it is settled true that") is the historical necessity operator, and [_ *dstit*: _] is the deliberative-seeing-to-it-that operator. The connectives $\top$, $\bot$, $\vee$, $\supset$, and $\equiv$ are defined as usual, $Poss : A$ abbreviates $\neg Sett : \neg A$, and the expressions $[\alpha \ cstit : A]$ and $\mathit{diff}(\alpha_0, \ldots, \alpha_n)$ are defined as follows.

DEFINITION 1. (1) $[\alpha\ cstit\colon A] :\equiv [\alpha\ dstit\colon A] \vee Sett\colon A$.
(2) $\mathit{diff}(\alpha_0) :\equiv \top$, and for $n \geq 0$, $\mathit{diff}(\alpha_0, \ldots, \alpha_{n+1}) :\equiv \mathit{diff}(\alpha_0, \ldots, \alpha_n) \wedge \neg(\alpha_0 = \alpha_{n+1}) \wedge \ldots \wedge \neg(\alpha_n = \alpha_{n+1})$.

A branching temporal structure with agents and choice is a quadruple $\mathfrak{S} = \langle Tree, \leq, Agent, Choice \rangle$, where $Tree$ is a non-empty set (of moments of time), and $\leq$ is a tree-like partial order on $Tree$. There is thus "historical connection" ($\forall m, m' \in Tree$, there exists $m'' \in Tree$ with $m'' \leq m$ and $m'' \leq m'$) and no "backward branching" ($\forall m_1, m_2, m_3 \in Tree$, if $m_1 \leq m_3$ and $m_2 \leq m_3$, then either $m_1 \leq m_2$ or $m_2 \leq m_1$). If $m_1 \leq m_2$ but not $m_2 \leq m_1$, we shall write $m_1 < m_2$. A maximal linearly ordered subset of $Tree$ is called a history in $\mathfrak{S}$. Intuitively, histories may be regarded as complete temporal developments of the world. In the present indeterministic branching time framework, a moment may belong to more than one history, and the set of histories to which a moment m belongs (that pass through the moment) is represented by $H_m := \{h \mid m \in h\}$. Given a branching temporal structure with agents and choice $\mathfrak{S} = \langle Tree, \leq, Agent, Choice \rangle$, $History$ is the set of all histories of $Tree$.

$Agent$ is a non-empty set (of agents), and $Choice$ is a function defined on $Agent \times Tree$ such that for every $x \in Agent$ and $m \in Tree$, $Choice(x, m)$, written as $Choice^x_m$, is a partition into equivalence classes of H_m. A model based on a $BT{+}AC$ structure $\mathfrak{S} = \langle Tree, \leq, Agent, Choice \rangle$ is a pair $\langle \mathfrak{S}, \mathfrak{I} \rangle$, where $\mathfrak{I}$ is a function defined on $Term \cup Atom$ such that for every agent term α and every $p \in Atom$, $\mathfrak{I}(\alpha) \in Agent$ and $\mathfrak{I}(p) \subseteq \{(m, h) \mid m \in Tree,\ \text{and}\ h \in H_m\}$. In the literature there is some discussion about which kinds of sentences ought to be semantically evaluated relative not only to moments of time but also relative to histories. It seems uncontroversial, however, that in an indeterministic semantical framework the truth conditions of contingent statements about the future are history-dependent. Also, the truth and non-truth of historical necessity statements depends on histories. As a matter of semantic uniformity, in dstit theory *every* formula is evaluated at pairs (m, h) consisting of a moment m and a history h passing through m.

If $h, h' \in H_m$ belong to the same set in $Choice^x_m$, we write $h \equiv^x_m h'$ and say that h and h' are choice-equivalent for x at m. The elements of $Choice^x_m$ are the choices (or choice cells) available to x at m, and x cannot distinguish by her or his actions at m between histories that are choice-equivalent for x at m. The set of histories choice equivalent with h for x at m is denoted by $Choice^x_m(h)$. The set of agents is held constant for the entire tree of moments.

The agents of a $BT + AC$ model are assumed to be independent of each other in the sense that at every moment, every agent must be able to realize any of her or his actions, no matter what choices are available to the other agents.

DEFINITION 2. Let $\mathfrak{S} = \langle Tree, \leq, Agent, Choice \rangle$ be a $BT + AC$ structure, and let $Select_m$ be the set of all functions s from $Agent$ into subsets of H_m, such that $s(x) \in Choice^x_m$. $\mathfrak{S}$ satisfies the *independence of agents* condition iff for every $m \in Tree$, $\bigcap_{x \in Agent} s(x) \neq \emptyset$ for every $s \in Select_m$.

Moreover, it is assumed that there is "no choice between undivided histories". Two histories $h, h' \in History$ are undivided at m ($h \equiv_m h'$) iff $\exists m'(m < m'$ and $m' \in h \cap h')$. $Choice$ is required to satisfy the following condition: $(\forall m \in Tree)$ $(\forall x \in Agent)$ $(\forall h, h' \in History)$ $h \equiv_m h'$ implies $h \equiv^x_m h'$.

DEFINITION 3. Let $\mathfrak{M} = \langle\langle Tree, \leq, Agent, Choice\rangle, \mathfrak{I}\rangle$ be a model. The truth of a formula A at a pair (m, h) from $\mathfrak{M}$ with $m \in h$ (in symbols $\mathfrak{M}, (m, h) \models A$) is inductively defined as follows:

$\mathfrak{M}, (m, h) \models p$	iff	$(m, h) \in \mathfrak{I}(p)$
$\mathfrak{M}, (m, h) \models (\alpha = \beta)$	iff	$\mathfrak{I}(\alpha) = \mathfrak{I}(\beta)$
$\mathfrak{M}, (m, h) \models \neg A$	iff	$\mathfrak{M} \not\models A$
$\mathfrak{M}, (m, h) \models (A \wedge B)$	iff	$\mathfrak{M}, (m, h) \models A$ and $\mathfrak{M}, (m, h) \models B$
$\mathfrak{M}, (m, h) \models Sett\colon A$	iff	$(\forall h' \in H_m)\, \mathfrak{M}, (m, h') \models A$
$\mathfrak{M}, (m, h) \models [\alpha\ dstit\colon A]$	iff	$(\forall h' \in Choice^{\mathfrak{I}(\alpha)}_m(h))\, \mathfrak{M}, (m, h') \models A$ and $(\exists h'' \in H_m)\, \mathfrak{M}, (m, h'') \not\models A$

We then have

$\mathfrak{M}, (m, h) \models Poss\colon A$	iff	$(\exists h' \in H_m)\, \mathfrak{M}, (m, h') \models A$
$\mathfrak{M}, (m, h) \models [\alpha\ cstit\colon A]$	iff	$(\forall h' \in Choice^{\mathfrak{I}(\alpha)}_m(h))\, \mathfrak{M}, (m, h') \models A$

Clearly, $Sett:$ and $[\_\ cstit : \_]$ are **S5**-type necessity operators: semantically, $Sett$: is a universal quantifier on H_m, and $[\_\ cstit\colon \_]$ is a universal quantifier on a subset of H_m.

DEFINITION 4. Let Δ be a set of formulas and A be a formula. $\Delta \models A$ (Δ entails A) iff for every model $\mathfrak{M} = \langle\langle Tree, \leq, Agent, Choice\rangle, \mathfrak{I}\rangle$ and every (m, h) with $m \in Tree$ and $h \in H_m$: if $\mathfrak{M}, (m, h) \models B$ for every $B \in \Delta$, then $\mathfrak{M}, (m, h) \models A$.

We shall refer to the logic $(\mathcal{L}, \models)$ as **Ldm**. This terminology is justified by the characterization result for the axiom system **Ldm** in [3, Chapter 17]. The axioms and inference rules (in addition to all substitution instances of classical tautologies) presented there are:

A1 $Sett\colon (A \supset B) \supset (Sett\colon A \supset Sett\colon B)$, $Sett\colon A \supset A$,
$Poss\colon A \supset Sett\colon Poss\colon A$

A2 $[\alpha\ cstit\colon (A \supset B)] \supset ([\alpha\ cstit\colon A] \supset [\alpha\ cstit\colon B])$, $[\alpha\ cstit\colon A] \supset A$, $\neg[\alpha\ cstit\colon A] \supset [\alpha\ cstit\colon \neg[\alpha\ cstit\colon A]]$

A3 $[\alpha\ dstit\colon A] \supset \neg Sett\colon A$

A4 $(\alpha = \alpha)$, $(\alpha = \beta) \supset (\beta = \alpha)$, $((\alpha = \beta) \wedge (\beta = \gamma)) \supset (\alpha = \gamma)$

A5 $(\alpha = \beta) \supset (A \supset A(\alpha/\beta))$, where $A(\alpha/\beta)$ is obtained from A by replacing some or all occurrences of α with β

AIA$_k$ $(\mathit{diff}(\beta_0, \dots, \beta_k) \wedge Poss\colon [\beta_0\ cstit\colon B_0] \wedge \dots \wedge Poss\colon [\beta_k\ cstit\colon B_k]) \supset Poss\colon ([\beta_0\ cstit\colon B_0] \wedge \dots \wedge [\beta_k\ cstit\colon B_k])$ $(k \geq 1)$

RN $\vdash A \,/\, \vdash Sett\colon A$

and *modus ponens*.

3 A tableau calculus for Ldm

We shall introduce a tableau calculus for **Ldm** in the style of the tableau systems presented in [11]. This format not only has found wide application, but also is particularly easy to grasp and admits of simple and very transparent soundness and completeness proofs. The tableau rules for **Ldm**, though perspicuous, are elaborate because of the richness of the semantics. The rules are used to build up counter-models for formulas and inferences that are not valid. Complete tableaux must (i) have a suitable tree structure, (ii) provide information about the histories passing through the moments of time, (iii) provide information about the choices available to the agents under consideration, and (iv) satisfy the independence of agents condition.

3.1 Tableau rules

If Δ is a set of formulas, then Δ^0 is defined as a set of certain compound expressions, namely $\Delta^0 := \{A, (m, h_0) \mid A \in \Delta\}$. A tableau is a rooted tree. The construction of a tableau starts with a set of expressions $\Delta^0 \cup \{\neg B, (m, h_0)\} \cup \{m \in h_0, m \lhd m_0, m_0 \in h_0\}$ to which identity rules, decomposition rules, and structural tableau rules may (or may not) be applied to obtain a tableau. The tableau nodes contain information about the model that is build up (in case the construction in fact yields a counter-model). The expression $m \lhd m_0$ gives the information that the considered moment m is earlier than m_0. Also information about certain relations between histories may be included. The expression $h_i \lhd^{\alpha}_{m} h_k$ indicates that h_i and h_k are choice-equivalent for α at m.

The set of tableau rules for **Ldm** is given in Tables 1–4. Decomposition rules are named after the kind of formula that is decomposed. The rule for negated conjunctions, for instance, is called $(\mathsf{R}\neg\wedge)$ and the rule for negated dstit formulas is called $(\mathsf{R}\neg[\ dstit\colon\ ])$. The indices i, j, k, l, n are elements from $\mathbb{N}$ (including 0), and a new index is the smallest natural number not already used in the tableau. If an expression is placed below an arrow, it is a rule output, otherwise it is a rule input. Note that the tableau rules

sub	ref	sym	tran
$A, (m, h_i)$ $\alpha = \beta$ $\downarrow$ $A(\alpha/\beta), (m, h_i)$	$\cdot$ $\downarrow$ $\alpha = \alpha$	$\alpha = \beta$ $\downarrow$ $\beta = \alpha$	$\alpha = \beta$ $\beta = \gamma$ $\downarrow$ $\alpha = \gamma$

Table 1. Identity rules

REF	SYM	TRAN
$\cdot$ $\downarrow$ $h_i \lhd_m^\alpha h_i$ for all terms α and every h_i on the tableau	$h_i \lhd_m^\alpha h_k$ $\downarrow$ $h_k \lhd_m^\alpha h_i$	$h_i \lhd_m^\alpha h_k$ $h_k \lhd_m^\alpha h_l$ $\downarrow$ $h_i \lhd_m^\alpha h_l$

IND

$$\begin{array}{c} h_{l_1} \lhd_m^{\alpha_1} h_{l_1} \ \dots \ h_{l_k} \lhd_m^{\alpha_k} h_{l_k} \\ \downarrow \\ m \lhd m_n \qquad\qquad n \text{ new} \\ m \in h_n,\ m_n \in h_n \\ h_{l_1} \lhd_m^{\alpha_1} h_n \ \dots \ h_{l_k} \lhd_m^{\alpha_k} h_n \end{array}$$

Table 2. Structural tableau rules

for which no input is stated may be applied to any node, that some rules introduce a new moment in order to create a new history, and that the left branch created by an application of (R$\neg$[*dstit*:]) must be further expanded by $A, (m, h_j)$ after the introduction of a new history h_j on that branch. A tableau is a tableau for $\Delta^0 \cup \{\neg B, (m, h_0)\}$ iff it is the result of applying some or no tableau rules to $\Delta^0 \cup \{\neg B, (m, h_0)\} \cup \{m \in h_0, m \lhd m_0, m_0 \in h_0\}$.

A tableau branch is said to be *closed* iff there are expressions of the form $A, (m, h_i)$ and $\neg A, (m, h_i)$ on the branch. A tableau is called *closed* iff all of its branches are closed. If a tableau (tableau branch) is not closed, it is called *open*. A tableau branch is said to be *complete* iff no more rules can be applied to expand it. A tableau is said to be *complete* iff each of its branches is complete. If a rule application yields an expression already on the branch, the rule is not applied. Moreover, we shall not apply the rule ref to introduce new agent terms not already on the tableau. The following definition is central.

DEFINITION 5. Let $\Delta \cup \{A\}$ be a set of $\mathcal{L}$-formulas. $\Delta \vdash A$ (A is derivable from Δ) iff there exists a closed and complete tableau for $\Delta^0 \cup \{\neg A, (m, h_0)\}$.

3.2 Examples

EXAMPLE 6. Note that we do not require a premise set to be finite. The reason is that a complete and closed tableau for a finite premise set Δ (a derivation from Δ) may already be infinite. The following simple example of an infinite open tableau is adapted from [11].

$$
\begin{array}{cl}
\neg\neg(\neg Sett\colon p \wedge Sett\colon \neg Sett\colon p),\ (m, h_0) & \\
m \lhd m_0,\ m \in h_0,\ m_0 \in h_0 & (\mathsf{R}\neg\neg) \\
\downarrow & \\
(\neg Sett\colon p \wedge Sett\colon \neg Sett\colon p),\ (m, h_0) & (\mathsf{R}\wedge) \\
\downarrow & \\
\neg Sett\colon p\ (m, h_0) & (\mathsf{R}\neg Sett\colon) \\
Sett\colon \neg Sett\colon p,\ (m, h_0) & \\
\downarrow & \\
m \lhd m_1,\ m \in h_1,\ m_1 \in h_1,\ \neg p, (m, h_1) & (\mathsf{R}Sett\colon) \\
\downarrow & \\
\neg Sett\colon p, (m, h_1) & (\mathsf{R}\neg Sett\colon) \\
\downarrow & \\
m \lhd m_2,\ m \in h_2,\ m_2 \in h_2,\ \neg p, (m, h_2) & (\mathsf{R}Sett\colon) \\
\downarrow & \\
\neg Sett\colon p, (m, h_2) & (\mathsf{R}\neg Sett\colon) \\
\downarrow & \\
\vdots &
\end{array}
$$

Next, we present some simple examples of finite closed tableaux.

EXAMPLE 7. We consider a formula logically equivalent with the negation of Axiom **A3** from [3, Chapter 17], see Section 2.

$$
\begin{array}{cl}
\neg\neg([\alpha\ dstit\colon p] \wedge Sett\colon p), (m, h_0) & \\
m \lhd m_0,\ m \in h_0,\ m_0 \in h_0 & (\mathsf{R}\neg\neg) \\
\downarrow & \\
([\alpha\ dstit\colon p] \wedge Sett\colon p), (m, h_0) & (\mathsf{R}\wedge) \\
\downarrow & \\
[\alpha\ dstit\colon p], (m, h_0) & \\
Sett\colon p, (m, h_0) & \mathsf{REF} \\
\downarrow & \\
h_0 \lhd^{\alpha}_{m} h_0 & (\mathsf{R}[\ dstit\colon]) \\
\downarrow & \\
p, (m, h_0) & \\
m \lhd m_1,\ m \in h_1,\ m_1 \in h_1 & (\mathsf{R}Sett\colon) \\
\neg p, (m, h_1) & \\
\downarrow & \\
p, (m, h_1) &
\end{array}
$$

EXAMPLE 8. We prove the symmetry axiom for historical necessity, i.e., we show that $\varnothing \vdash \neg(p \wedge \neg Sett\colon \neg Sett\colon \neg p)$.

$$\begin{array}{c} \neg\neg A, (m, h_i) \\ \downarrow \\ A, (m, h_i) \end{array}$$

$$\begin{array}{c} (A \wedge B), (m, h_i) \\ \downarrow \\ A, (m, h_i) \\ B, (m, h_i) \end{array} \qquad \begin{array}{c} \neg(A \wedge B), (m, h_i) \\ \swarrow \quad \searrow \\ \neg A, (m, h_i) \quad \neg B, (m, h_i) \end{array}$$

$$\begin{array}{c} Sett\colon A, (m, h_i) \\ m \in h_k \\ \downarrow \\ A, (m, h_k) \end{array} \qquad \begin{array}{c} \neg Sett\colon A, (m, h_i) \\ \downarrow \\ m \lhd m_k \quad k \text{ new} \\ m \in h_k,\ m_k \in h_k \\ \neg A, (m, h_k) \end{array}$$

$$\begin{array}{c} [\alpha\ dstit\colon A], (m, h_i) \\ h_i \lhd_m^\alpha h_k \\ \downarrow \\ A, (m, h_k) \\ m \lhd m_l \quad l \text{ new} \\ m \in h_l,\ m_l \in h_l \\ \neg A, (m, h_l) \end{array} \qquad \begin{array}{c} \neg[\alpha\ dstit\colon A], (m, h_i) \\ \swarrow \qquad \searrow \\ \begin{array}{c} A, (m, h_l) \\ \text{for every } h_l \text{ with} \\ m \in h_l \text{ on the} \\ \text{branch} \end{array} \quad \begin{array}{c} m \lhd m_k \quad k \text{ new} \\ m \in h_k,\ m_k \in h_k \\ h_i \lhd_m^\alpha h_k \\ \neg A, (m, h_k) \end{array} \end{array}$$

Table 3. Decomposition rules for **Ldm**

$$\begin{array}{cl} \neg\neg(p \wedge \neg Sett\colon \neg Sett\colon \neg p), (m, h_0) & \\ m \lhd m_0,\ m \in h_0,\ m_0 \in h_0 & (\mathsf{R}\neg\neg) \\ \downarrow & \\ (p \wedge \neg Sett\colon \neg Sett\colon \neg p), (m, h_0) & (\mathsf{R}\wedge) \\ \downarrow & \\ p, (m, h_0) & \\ \neg Sett\colon \neg Sett\colon \neg p, (m, h_0) & (\mathsf{R}\neg Sett\colon) \\ \downarrow & \\ m \lhd m_1,\ m \in h_1,\ m_1 \in h_1 & \\ \neg\neg Sett\colon \neg p, (m, h_1) & (\mathsf{R}\neg\neg) \\ \downarrow & \\ Sett\colon \neg p, (m, h_1) & (\mathsf{R}Sett\colon) \\ \downarrow & \\ \neg p, (m, h_0) & \end{array}$$

EXAMPLE 9. The axioms AIA_k are used in [3] to prove that the canonical model for **Ldm** satisfies the independence of agents condition. We consider a formula logically equivalent with the negation of the result of dropping one conjunct from the antecedent of an instance of AIA_k. The omitted conjunct is $\neg(\alpha_1 = \alpha_2)$. It is satisfied in our tableau if we assume that the agent terms α_1 and α_2 are uniquely denoting terms, see also Definition 13, where we define $\mathfrak{I}(\alpha) := \alpha$.

$$
\begin{array}{c}
\neg\neg((Poss\,[\alpha_1\ cstit\colon p_1] \wedge Poss\,[\alpha_2\ cstit\colon p_2])\wedge \\
\neg Poss\,([\alpha_1\ cstit\colon p_1] \wedge [\alpha_2\ cstit\colon p_2])),\ (m, h_0) \\
m \lhd m_0,\ m \in h_0,\ m_0 \in h_0 \\
\downarrow \\
((Poss\,[\alpha_1\ cstit\colon p_1] \wedge Poss\,[\alpha_2\ cstit\colon p_2])\wedge \\
\neg Poss\,([\alpha_1\ cstit\colon p_1] \wedge [\alpha_2\ cstit\colon p_2])),\ (m, h_0) \\
\downarrow \\
(Poss\,[\alpha_1\ cstit\colon p_1] \wedge Poss\,[\alpha_2\ cstit\colon p_2]),\ (m, h_0) \\
\neg Poss\,([\alpha_1\ cstit\colon p_1] \wedge [\alpha_2\ cstit\colon p_2]),\ (m, h_0) \\
\downarrow \\
Poss\,[\alpha_1\ cstit\colon p_1],\ (m, h_0) \\
Poss\,[\alpha_2\ cstit\colon p_2],\ (m, h_0) \\
\downarrow \\
m \lhd m_1,\ m \in h_1,\ m_1 \in h_1 \\
[\alpha_1\ cstit\colon p_1],\ (m, h_1) \\
\downarrow \\
m \lhd m_2,\ m \in h_2,\ m_2 \in h_2 \\
[\alpha_2\ cstit\colon p_2],\ (m, h_2) \\
\downarrow \\
h_1 \lhd_m^{\alpha_1} h_1 \\
\downarrow \\
h_2 \lhd_m^{\alpha_2} h_2 \qquad\qquad\qquad\qquad \mathsf{IND} \\
\downarrow \\
m \lhd m_3,\ m \in h_3,\ m_3 \in h_3 \\
h_1 \lhd_m^{\alpha_1} h_3 \\
h_2 \lhd_m^{\alpha_2} h_3 \\
\downarrow \\
p_1, (m, h_3) \\
\downarrow \\
p_2, (m, h_3) \\
\downarrow \\
\neg([\alpha_1\ cstit\colon p_1] \wedge [\alpha_2\ cstit\colon p_2]),\ (m, h_3) \\
\swarrow \qquad \searrow
\end{array}
$$

$$
\begin{array}{cc}
\neg[\alpha_1\ cstit\colon p_1],\ (m, h_3) & \neg[\alpha_2\ cstit\colon p_2]),\ (m, h_3) \\
\downarrow & \downarrow \\
m \lhd m_4,\ m \in h_4,\ m_4 \in h_4, & m \lhd m_5,\ m \in h_5,\ m_5 \in h_5, \\
\neg p_1,\ (m, h_4) & \neg p_2,\ (m, h_5) \\
\downarrow & \downarrow \\
p_1,\ (m, h_4) & p_2,\ (m, h_5)
\end{array}
$$

4 Soundness and completeness

We show that for every set of $\mathcal{L}$-formulas $\Delta \cup \{A\}$, $\Delta \models A$ iff $\Delta \vdash A$. We first prove the right-to-left direction of this characterization of **Ldm**, i.e., soundness.

DEFINITION 10. Let $\mathfrak{M} = \langle\langle Tree, \leq, Agent, Choice\rangle, \mathfrak{I}\rangle$ be a model, and b be a tableau branch. The model $\mathfrak{M}$ is faithful to b iff there exists a function,

$$\begin{array}{ccc} \begin{array}{c} Poss\colon A, (m, h_i) \\ \downarrow \\ m \lhd m_k \\ m \in h_k,\ m_k \in h_k \\ A, (m, h_k) \end{array} & \begin{array}{c} \\ \\ k \text{ new} \\ \\ \\ \end{array} & \begin{array}{c} \neg Poss\colon A, (m, h_i) \\ m \in h_k \\ \downarrow \\ \neg A, (m, h_k) \\ \\ \end{array} \end{array}$$

$$\begin{array}{cc} \begin{array}{c} [\alpha\ cstit\colon A], (m, h_i) \\ h_i \lhd^\alpha_m h_k \\ \downarrow \\ A, (m, h_k) \\ \\ \\ \end{array} & \begin{array}{c} \neg[\alpha\ cstit\colon A], (m, h_i) \\ \downarrow \\ m \lhd m_k \qquad k \text{ new} \\ m \in h_k,\ m_k \in h_k \\ h_i \lhd^\alpha_m h_k \\ \neg A, (m, h_k) \end{array} \end{array}$$

Table 4. Derived decomposition rules

f, from $\{m\} \cup \{h_i \mid i \in \mathbb{N}\}$ into $Tree \cup History$ such that:

1. For every expression $A, (m, h_i)$ on b, $\mathfrak{M}, (f(m), f(h_i)) \models A$.
2. If $m \in h_k$ occurs on b, then $f(h_k) \in H_{f(m)}$.
3. If $h_i \lhd^\alpha_m h_k$ occurs on b, then $f(h_k) \in Choice^{\mathfrak{I}(\alpha)}_{f(m)}(f(h_i))$.

The function f is said to show that $\mathfrak{M}$ is faithful to branch b.

LEMMA 11. *Let $\mathfrak{M} = \langle\langle Tree, \leq, Agent, Choice\rangle, \mathfrak{I}\rangle$ be a model, and b be a tableau branch. If $\mathfrak{M}$ is faithful to b and a tableau rule is applied to b, then the application produces at least one extension b' of b, such that $\mathfrak{M}$ is faithful to b'.*

Proof. Assume that f is a function that shows $\mathfrak{M}$ to be faithful to b. We have to consider each of the tableau rules. If the extended branch b' is obtained by applying an identity rule or one of the rules for $\neg\neg A$, $(A \wedge B)$ or $\neg(A \wedge B)$, obviously f shows $\mathfrak{M}$ to be faithful to b'. (For sub we have to use induction on the complexity of A.) Suppose the rule for $Sett\colon A$ is applied to $Sett\colon A, (m, h_i)$ and $m \in h_k$ to obtain $b' = b$ extended with $A, (m, h_k)$. Since f is faithful to b, we have $\mathfrak{M}, (f(m), f(h_i)) \models Sett : A$. Therefore, $(\forall h' \in H_{f(m)})\ \mathfrak{M}, (f(m), h') \models A$. By assumption, we also have $f(h_k) \in H_{f(m)}$. Thus, $\mathfrak{M}, (f(m), f(h_k)) \models A$ and f shows $\mathfrak{M}$ to be faithful to b'. Suppose now the rule for $\neg Sett\colon A$ is applied to $\neg Sett\colon A, (m, h_i)$, so that b is extended by $m \lhd m_k$, $m \in h_k$, $m_k \in h_k$, and $\neg A, (m, h_k)$, for a new index k. Since f shows $\mathfrak{M}$ to be faithful to b, we have $\mathfrak{M}, (f(m), f(h_i)) \models \neg Sett\colon A$. Therefore, $(\exists h' \in H_{f(m)})\ \mathfrak{M}, (f(m), h') \models \neg A$. Define f' to be the same function as f, except that $f'(h_k) = h'$. Then $\mathfrak{M}, (f(m), f'(h_k)) \models \neg A$ and $f'(h_k) \in H_{f(m)}$. The function f' shows $\mathfrak{M}$ to be faithful to the extended branch b'. Next, assume that the rule for $\neg[\alpha\ dstit : A]$ is applied to

$\neg[\alpha\ dstit: A], (m, h_i)$ to obtain branch $b' = b$ extended by $A, (m, h_j)$ for every expression $m \in h_j$ on b and branch $b'' = b$ extended by $m \lhd m_k$, $m \in h_k$, $m_k \in h_k$, $h_i \lhd^{\alpha}_{m} h_k$ and $\neg A, (m, h_k)$, for a new index k. Since f shows $\mathfrak{M}$ to be faithful to b, we know that $\mathfrak{M}, (f(m), f(h_i)) \models \neg[\alpha\ dstit: A]$. Hence either (a) $(\exists h' \in Choice^{\Im(\alpha)}_{f(m)}(f(h_i)))\ \mathfrak{M}, (f(m), h') \models \neg A$ or (b) $(\forall h'' \in H_{f(m)})\ \mathfrak{M}, (f(m), h'') \models A$. Consider case (a). Let f' be the same function as f, except that $f'(h_k) = h'$. Then $\mathfrak{M}, (f(m), f'(h_k)) \models \neg A$. Since $f'(h_k) = h' \in Choice^{\Im(\alpha)}_{f(m)}(f(h_i)) \subseteq H_{f(m)}$, $f'(h_k) \in H_{f(m)}$. The function f' shows $\mathfrak{M}$ to be faithful to b''. Consider case (b). For every expression $m \in h_j$ on b, $f(h_j) \in H_{f(m)}$. Thus, for every expression $m \in h_j$ on b, $\mathfrak{M}, (f(m), f(h_j)) \models A$. Therefore, f shows $\mathfrak{M}$ to be faithful to b'. The last decomposition rule to inspect is the rule for $[\alpha\ dstit: A]$. Suppose the rule is applied to $[\alpha\ dstit: A], (m, h_i)$ and $h_i \lhd^{\alpha}_{m} h_k$ to obtain $A, (m, h_k)$, $m \lhd m_l$, $m \in h_l$, $m_l \in h_l$, and $\neg A, (m, h_l)$, for some new index l. As f shows $\mathfrak{M}$ to be faithful to b, we have $\mathfrak{M}, (f(m), f(h_i)) \models [\alpha\ dstit: A]$, which means that $(\forall h' \in Choice^{\Im(\alpha)}_{f(m)}(f(h_i)))\ \mathfrak{M}, (f(m), h') \models A$ and $(\exists h'' \in H_{f(m)})\ \mathfrak{M}, (f(m), h'') \models \neg A$. Let f' be the same as f, except that $f'(h_l) = h''$. Then $\mathfrak{M}, (f(m), f(h_l)) \models \neg A$ and $f(h_l) \in H_{f(m)}$. Since by assumption $f(h_k) \in Choice^{\Im(\alpha)}_{f(m)}(f(h_i)))$, also $\mathfrak{M}, (f(m), f'(h_k)) \models A$, and thus f' shows $\mathfrak{M}$ to be faithful to b'. It remains to consider the structural tableau rules. The first three structural rules satisfy the condition, because for every agent x and every moment $m \in Tree$, the relation $\equiv^{x}_{m}$ is an equivalence relation. For IND, we have $f(h_n) \in Choice^{\alpha_1}_{f(m)}(f(h_{l_1}))\ \ldots$ $f(h_n) \in Choice^{\alpha_k}_{f(m)}(f(h_{l_k}))$, because $\mathfrak{M}$ satisfies the independence of agents condition. Moreover, since $f(h_n) \in Choice^{\alpha_1}_{f(m)}(f(h_{l_1})) \subseteq H_{f(m)}$, $f(h_n) \in H_{f(m)}$. ∎

THEOREM 12. *If* $\Delta \not\models A$, *then* $\Delta \not\vdash A$.

Proof. If $\Delta \not\models A$, then there is a model $\mathfrak{M}$ and a moment-history pair (m', h) from this model, such that for every $B \in \Delta$, $\mathfrak{M}, (m', h) \models B$, but $\mathfrak{M}, (m', h) \not\models A$. Let f be a function from $\{m\} \cup \{h_i \mid i \in \mathbb{N}\}$ into $Tree \cup History$, such that $f(m) = m'$ and $f(h_0) = h$. Then f shows that $\mathfrak{M}$ is faithful to $\Delta^0 \cup \{\neg A, (m, h_0)\} \cup \{m \in h_0, m \lhd m_0, m_0 \in h_0\}$. By the previous lemma, for any tableau for $\Delta^0 \cup \{\neg A, (m, h_0)\}$ and hence for any complete tableau for $\Delta^0 \cup \{\neg A, (m, h_0)\}$ there exists a branch b such that f is faithful to every initial segment of b. Such a branch b cannot be closed, because otherwise there are formulas A and $\neg A$ and a moment-history pair (m, h''), such that $\mathfrak{M}, (m, h'') \models A \land \neg A$. Hence there exists no closed and complete tableau for $\Delta^0 \cup \{\neg A, (m, h_0)\}$. ∎

We now prove the left-to-right direction of the characterization of **Ldm**, i.e., completeness.

DEFINITION 13. Let b be an open branch of a complete tableau. The model $\mathfrak{M}_b = \langle\langle Tree, \leq, Agent, Choice\rangle, \Im\rangle$ induced by b is defined as follows:

- $Tree := \{m\} \cup \{m_i \mid m_i \text{ occurs on } b\}$.
- $\leq :=$ the reflexive (and transitive) closure of $\{(m, m_j) \mid m \lhd m_j \text{ on } b\}$.
- $Agent := \{\alpha \mid \alpha \text{ is an agent term occurring on } b\}$.
- $Choice^{\alpha}_{m}(h_k) := \{h_l \mid h_k \lhd^{\alpha}_{m} h_l \text{ occurs on } b\}$.
- $\mathfrak{I}(\alpha) := \alpha$.

Moreover, the definition of $\mathfrak{I}$ for atomic formulas p is arbitrary, except that we stipulate:

- $(m, h_k) \in \mathfrak{I}(p)$ if $p, (m, h_k)$ occurs on b.
- $(m, h_k) \notin \mathfrak{I}(p)$ if $\neg p, (m, h_k)$ occurs on b.

The model $\mathfrak{M}_b$ is well-defined, because b is open. There exists no pair (m, h_i) and no $p \in Atom$, such that $(m, h_i) \in \mathfrak{I}(p)$ and $(m, h_i) \notin \mathfrak{I}(p)$. Note that we have $H_m = \{h_k \mid m \in h_k \text{ on } b\}$. By the tableau rules introducing expressions $m \lhd m_j$ it is clear that $\leq$ is a tree-like partial order on $Tree$. Since b is complete, by the structural rules REF, SYM, and TRAN it is clear that $Choice^{\alpha}_{m}$ is a partition of H_m and by the structural rule IND it is clear that the independence of agents condition is satisfied. For any moment m and any finite set of histories $h_1, \ldots, h_n$, each belonging to a choice cell of pairwise distinct agents $\alpha_1, \ldots, \alpha_n$, there exists a history $h \in H_m$ such that $h \in Choice^{\alpha_1}_{m}(h_1) \cap \ldots \cap Choice^{\alpha_n}_{m}(h_n)$. Clearly, the tableau construction also guarantees that the "no choice between undivided histories" condition is satisfied.

LEMMA 14. *Let b be an open branch of a complete tableau, and let $\mathfrak{M}_b = \langle\langle Tree, \leq, Agent, Choice\rangle, \mathfrak{I}\rangle$ be the model induced by b. Then*

1. *if $A, (m, h_i)$ occurs on b, then $\mathfrak{M}_b, (m, h_i) \models A$;*
2. *if $\neg A, (m, h_i)$ occurs on b, then $\mathfrak{M}_b, (m, h_i) \not\models A$.*

Proof. By simultaneous induction on the complexity of A. For atoms the claims hold by definition. Assume $A = \neg B$. 2. If $\neg\neg B, (m, h_i)$ occurs on b, then $B, (m, h_i)$ occurs on b and by the induction hypothesis for 1., $\mathfrak{M}_b, (m, h_i) \models B$ and hence $\mathfrak{M}_b, (m, h_i) \not\models \neg B$. 1. By induction on B. It is enough to consider the second claim for the other inductive cases. $A = (B \wedge C)$. Use the induction hypotheses. $A = Sett : B$. 1. If $Sett : B, (m, h_i)$ is on b, then for every h_k such that $m \in h_k$ occurs on b, we have it that $B, (m, h_k)$ occurs on b. By the induction hypothesis for 1. and the definition of $\mathfrak{M}_b$: $\mathfrak{M}_b, (m, h_i) \models Sett : B$. 2. If $\neg Sett : B, (m, h_i)$ occurs on b, then there exists an index k such that $m \in h_k$ and $\neg B, (m, h_k)$ occur on b. By the induction hypothesis for 2. and the definition of $\mathfrak{M}_b$: $\mathfrak{M}_b, (m, h_i) \models \neg Sett : B$. $A = [\alpha\ dstit\colon B]$. 1. If $[\alpha\ dstit\colon B], (m, h_i)$ occurs on b, then for every h_k such that $h_i \lhd^{\alpha}_{m} h_k$ occurs on b, also $B, (m, h_k)$

occurs on b. Moreover, there exists a history h_l such that $m \in h_l$ and $\neg B, (m, h_l)$ occur on b. By the induction hypotheses for 1. and 2. and the definition of $\mathfrak{M}_b$: $\mathfrak{M}_b, (m, h_i) \models [\alpha\ dstit\colon B]$. 2. If $\neg[\alpha\ dstit\colon B], (m, h_i)$ is on b, then either there exists h_k such that $\neg B, (m, h_k)$ and $h_i \lhd_m^\alpha k_k$ occur on b, or for every h_l such that $m \in h_l$ occurs on b, also $B, (m, h_l)$ is on b. By the definition of $\mathfrak{M}_b$, $h_k \in Choice_m^\alpha(h_i)$ and therefore, by the induction hypotheses for 1. and 2., $\mathfrak{M}_b, (m, h_i) \models \neg[\alpha\ dstit\colon B]$. ■

THEOREM 15. *If* $\Delta \nvdash A$, *then* $\Delta \not\models A$.

Proof. Suppose that $\Delta \nvdash A$, and let b be an open branch of a tableau for $\Delta^0 \cup \{\neg A, (m, h_0)\}$. Then for every formula $B \in \Delta$: $\mathfrak{M}_b, (m, h_0) \models B$, whereas $\mathfrak{M}_b, (m, h_0) \not\models A$. Hence $\Delta \not\models A$. ■

COROLLARY 16. *If one complete tableau for* $\Delta^0 \cup \{\neg A, (m, h_0)\}$ *is open, then every complete tableau for* $\Delta^0 \cup \{\neg A, (m, h_0)\}$ *is open.*

Proof. Suppose there is both (i) a complete and open tableau for $\Delta^0 \cup \{\neg A, (m, h_0)\}$ and (ii) a complete and closed tableau for $\Delta^0 \cup \{\neg A, (m, h_0)\}$. From (i) it follows by Lemma 14 that $\Delta \not\models A$ and from (ii) it follows by soundness that $\Delta \models A$: a contradiction. ■

From Corollary 16 it follows that in principle the order of tableau rule applications is irrelevant for proof search. For practical purposes, however, it may be useful to postpone the application of rules that introduce branching or new moments and histories as far as possible. It is known that **Ldm** is decidable [3, 18]. Note, however, that the present tableau calculus is not tailored for defining tableau algorithms. Nevertheless, when it comes to proof search the tableau calculus may be seen to be an improvement in comparison with axiomatic systems, because the decomposition rules provide heuristics.

5 Further directions

5.1 Introducing temporal operators

In [3, p. 435] Belnap, Perloff, and Xu explain that it

> *is surely very natural to combine dstit theory with indeterminist tense logics, especially when we consider* deliberative seeing to something *to be connected with what the future will be like. In carrying out some basic technical work in dstit theory, however, we will use a formal language without tense operators, though we will use the historical necessity operator … as a primitive.*

A first step towards introducing tense operators might be considering the Priorean temporal operators $\langle F\rangle$ ("sometimes in the future") and $\langle P\rangle$ ("sometimes in the past") to obtain the extended language $\mathcal{L}'$. The dual operators $[F]$ ("always in the future") and $[P]$ ("always in the past") are defined as usual: $[F]A := \neg\langle F\rangle\neg A$ and $[P]A := \neg\langle P\rangle\neg A$. The semantics of $\langle F\rangle$ and $\langle P\rangle$ is given with the following evaluation clauses.

DEFINITION 17. Let $\mathfrak{M} = \langle\langle Tree, \leq, Agent, Choice\rangle, \mathfrak{I}\rangle$ be a model, $m \in Tree$, and $h \in H_m$.

$$\mathfrak{M}, (m,h) \models \langle F\rangle A \quad \text{iff} \quad (\exists m' \in Tree)\ m < m' \text{ and } \mathfrak{M}, (m',h) \models A$$

$$\mathfrak{M}, (m,h) \models \langle P\rangle A \quad \text{iff} \quad (\exists m' \in Tree)\ m' < m \text{ and } \mathfrak{M}, (m',h) \models A$$

Entailment of a formula A by a set of formulas Δ ($\Delta \models A$) is defined as before, and the logic $(\mathcal{L}', \models)$ obtained in this way may be called **Ldmt**.

In order to use the previous definition of a single-conclusion derivability relation $\vdash$ for the extended language, we have to define tableau rules for the Priorean operators. These tableau rules must manipulate expressions $A, (m_i, h_k)$ instead of $A, (m, h_i)$, because now also the moment of evaluation and not only the history of evaluation may vary. Decomposition rules for temporal $\mathcal{L}'$-formulas that come to mind are:

$$\begin{array}{ccc} \langle F\rangle A, (m_i, h_k) & & \neg\langle F\rangle A, (m_i, h_k) \\ \downarrow & & m_i \lhd m_l \\ m_n \lhd m_l & l \text{ new} & m_l \in h_k \\ m_l \in h_k & n \text{ greatest index with} & \downarrow \\ A, (m_l, h_k) & m_n \in h_k \text{ already on tableau} & \neg A, (m_l, h_k) \end{array}$$

$$\begin{array}{ccc} \langle P\rangle A, (m_i, h_k) & & \neg\langle P\rangle A, (m_i, h_k) \\ \downarrow & & m_l \lhd m_i \\ m_{r-1} \lhd m_r & r \text{ smallest index on tableau} & \downarrow \\ A, (m_{r-1}, h_k) & & \neg A, (m_l, h_k) \end{array}$$

It is not obvious, however, how the method of the present paper could be applied to obtain a characterization result for **Ldmt**. A function f showing a model $\mathfrak{M}$ to be faithful to a tableau branch b would have to be a function into $Tree \cup History$ and would have to satisfy the condition that if $m_i \lhd m_l$ occurs on b, then $f(m_i) < f(m_l)$, in order to deal with applications of the rule for $\neg\langle F\rangle A$, (m_i, h_k). But it is not clear how this constraint can be satisfied for applications of the rule for $\neg Sett: A$, (m_i, h_k), for instance. The latter rule requires the definition of a function f' and introduces a new moment that does *not* become a moment of evaluation. We leave it as an open problem to define a sound and complete tableau calculus for **Ldmt**.

5.2 Beliefs, desires, intentions, and actions

A very prominent and widely accepted and applied approach to reasoning about rational agents is the belief-desire-intention (**BDI**) model, based on Bratman's theory of rational agency [4]. In **BDI** logics, the intentions of an agent are represented by sets of worlds "the agent has *chosen* to attempt to realize" [12, p. 304]. It is not required that intention operators of the form

INTEND_α (agent α intends that) are applied only to sentences reporting an action of α. Actually, the language of **BDI** logics as presented in [12] does not comprise any action modalities at all. It seems, however, that this syntax is not fully supported by natural language linguistic data.

Consider the following expressions:

(1) α intends to visit Berlin.

(2) α intends that α visits Berlin.

(3) α intends that α sees to it that α visits Berlin.

(4) α intends that the sun will rise tomorrow.

(5) α intends that $2 + 2 = 4$.

(6) α intends that β visits Berlin.

(7) α intends that α sees to it that β visits Berlin.

(8) α intends to intend to visit Berlin.

(9) α intends that α intends to visit Berlin.

(10) α intends that α intends that α visits Berlin.

(11) α sees to it that α intends to visit Berlin.

Although the verb ‘to intend’ may take ‘that’-clauses, its primary use in the lexical meaning of ‘to purpose’ is in sentences like (1), which may be understood as (2) or as (3). The intentions of an agent α, it seems, are directed towards α’s present or future actions or α’s participation in present or future joint actions. Therefore, sentences (4) and (5) are strange. In making a plan, an agent α may take it for granted that the sun will rise tomorrow or that $2 + 2 = 4$, but α cannot purpose that the sun will rise or that $2 + 2 = 4$, because the expressions ‘the sun will rise tomorrow’ and ‘$2 + 2 = 4$’ fail to describe an action of α. Can the intention of an agent α be directed towards *another* agents’s present or future actions? Of course, an agent α may take into account or assume in her or his plans that another agent visits Berlin, but it seems that α’s intentions to act indeed pertain to α’s actions or actions α participates in. In this respect, sentence (6) is strange, because the complement of the ‘that’-clause does not ascribe an action to α. One might thus wonder whether a sentence such as (7), which, intuitively, differs in meaning form (6), is satisfiable. If we assume that β is an agent not only distinct from α but also *independent of* α, it seems natural also to suppose that α cannot, in fact, guarantee by her or his actions that β (deliberatively sees to it that β) visits Berlin. This intuition about independent agents is properly reflected by the fact that nested other-agent dstit-formulas $[\alpha\ dstit\colon [\beta\ dstit\colon A]]$ are unsatisfiable for distinct agents α and β satisfying the independence of agents condition, see [17] and [3].

Assuming that (1)–(3) are synonymous, we may take 'α intends that α sees to it that A' to display a canonical form of intention ascriptions. The idea is to restrict the complement of 'α intends that' to sentences reporting a (present or future) action of α. This immediately leads to asking whether sentences like (8)–(10) are interpretable. The question with respect to (9), for instance, is not whether in α's plans α assumes that α intends to visit Berlin, but whether 'α intends to visit Berlin' is agentive for α, i.e., whether the latter sentence reports an action of α. In stit theory it has been suggested that a sentence A is agentive for α just in case A "may be usefully paraphrased as" 'α sees to it that A'; this is the so-called Stit paraphrase thesis, [3, §1]. The sentence 'α is in Berlin' cannot usefully be paraphrased as 'α sees to it that α is in Berlin', but the sentence 'α closes the door' can be usefully paraphrased as 'α sees to it that α closes the door'. Can (1) be usefully paraphrased as (11)? We might have the idea that *forming* an intention is an action type, but forming an intention is to be distinguished from *having* an intention. If intentions are regarded as mental states that have propositions as their contents, and if intending is thought of as a propositional attitude between a single or collective agent and a proposition, then the Stit paraphrase thesis seems not to apply to sentences (1) and (11). Agent α sees to it that α visits Berlin just in case α visits Berlin, but it is far from clear that α sees to it that α intends to visit Berlin if and only if α intends to visit Berlin. Moreover, the following discourse is infelicitous:

(12) α: What are you doing at the moment?
β: I am intending to visit Berlin.

What we are suggesting is a commitment to a specific version of stit theory's

Restricted complement Thesis. *A variety of constructions concerned with agents and agency—including deontic statements, imperatives, and statements of intention, among others—must take agentives as* their *complements.* [3, p. 13].

Namely, the idea is to modify the syntax of **BDI** logics by requiring that if A is a formula and α an agent term, then $[\alpha\ dstit\colon A]$ and $\mathsf{INTEND}_\alpha[\alpha\ dstit\colon A]$ are formulas. Note that we admit the formation of formulas $[\alpha\ dstit : \mathsf{INTEND}_\alpha[\alpha\ dstit\colon A]]$. An agent's formation of an intention may thereby be reported as an action.

In the relational models of **BDI** logics it is assumed that possible worlds are branching temporal structures (without agents and choice).

DEFINITION 18. A **BDI**-frame is a structure $\langle T, R, W, Agent, \{\mathcal{B}_x \mid x \in Agent\}, \{\mathcal{D}_x \mid x \in Agent\}, \{\mathcal{I}_x \mid x \in Agent\}\rangle$, where T is a non-empty set of moments, $R \subseteq T \times T$, $W = \{\langle T', R'\rangle \mid \varnothing \neq T' \subseteq T, R' \subseteq R, R'$ is a branching temporal structure over $T'\}$, $\mathcal{B}_x \subseteq W \times T \times W$, $\mathcal{D}_x \subseteq W \times T \times W$, $\mathcal{I}_x \subseteq W \times T \times W$. A pair (w, m) with $w = \langle T', R' \rangle$ and $m \in T'$ is called a situation.

Intuitively, the relation $\mathcal{B}_x$ ($\mathcal{D}_x$, $\mathcal{I}_x$) gives the set of *worlds* compatible with what agent x believes (desires, intends) at a situation, and usually one would assume that $\mathcal{B}_x$ is serial, transitive, and Euclidean and that $\mathcal{D}_x$ and $\mathcal{I}_x$ are serial. Formulas are evaluated in models based on **BDI**-frames at situations. The evaluation clauses of interest for our present considerations are:

$$
\begin{array}{lll}
(w,m) \models \mathsf{BEL}_\alpha A & \text{iff} & \forall w' \, ((w,m)\, \mathcal{B}_{\Im(\alpha)}\, w' \;\Rightarrow\; (w',m) \models A) \\
(w,m) \models \mathsf{DES}_\alpha A & \text{iff} & \forall w' \, ((w,m)\, \mathcal{D}_{\Im(\alpha)}\, w' \;\Rightarrow\; (w',m) \models A) \\
(w,m) \models \mathsf{INTEND}_\alpha A & \text{iff} & \forall w' \, ((w,m)\, \mathcal{I}_{\Im(\alpha)}\, w' \;\Rightarrow\; (w',m) \models A)
\end{array}
$$

Let $H_{w,m}$ be the set of histories passing through m in world w, and let $T_w := \{m \mid w = \langle T', R' \rangle \text{ and } m \in T'\}$. Instead of directly representing intentions by a relation between an agent α and α's future and present actions, we suggest to represent intentions by a relation between an agent α and propositions expressed by formulas agentive in α. This can be realized by evaluating formulas not in situations (w, m), but in triples (w, m, h), where $h \in H_{w,m}$.

DEFINITION 19. A **BDIA**-frame is a tuple $\langle T, R, W, Agent, \{Choice_w \mid w \in W\}, \{\mathcal{B}_x \mid x \in Agent\}, \{\mathcal{D}_x \mid x \in Agent\}, \{\mathcal{I}_x \mid x \in Agent\}\rangle$, where $\langle T, R, W, Agent, \{\mathcal{B}_x \mid x \in Agent\}, \{\mathcal{D}_x \mid x \in Agent\}, \{\mathcal{I}_x \mid x \in Agent\}\rangle$ is a **BDI**-frame and $Choice_w$ is a function defined on $Agent \times T_w$ such that for every $x \in Agent$ and $m \in T_w$, $Choice_w(x, m)$, written as $Choice^x_{w,m}$, is a partition into equivalence classes of $H_{w,m}$.

We may then use the following evaluation clauses in models based on **BDIA**-frames:

$$
\begin{array}{lll}
(w,m,h) \models \mathsf{BEL}_\alpha A & \text{iff} & \forall w' \, ((w,m)\, \mathcal{B}_{\Im(\alpha)}\, w' \;\Rightarrow\; (w',m,h) \models A) \\
(w,m,h) \models \mathsf{DES}_\alpha A & \text{iff} & \forall w' \, ((w,m)\, \mathcal{D}_{\Im(\alpha)}\, w' \;\Rightarrow\; (w',m,h) \models A) \\
(w,m,h) \models \mathsf{INTEND}_\alpha A & \text{iff} & \forall w' \, ((w,m)\, \mathcal{I}_{\Im(\alpha)}\, w' \;\Rightarrow\; (w',m,h) \models A)
\end{array}
$$

$$
\begin{array}{lll}
(w,m,h) \models [\alpha\ dstit\colon A] & \text{iff} & (\forall h' \in Choice^{\Im(\alpha)}_{w,m}(h))\ (w,m,h') \models A \text{ and} \\
& & (\exists h'' \in H_{w,m})\ (w,m,h'') \not\models A
\end{array}
$$

Another and, perhaps, conceptually simpler way of obtaining a combined modal logic of beliefs, desires, intentions, and actions is introducing belief, desire, and intention modalities into dstit logic along the lines suggested in [15, 16, 17]. We stipulate that if A is a formula and α an agent term, then $\mathsf{BEL}_\alpha A$, $\mathsf{DES}_\alpha A$ and $\mathsf{INTEND}_\alpha[\alpha\ dstit\colon A]$ are formulas. An extended model based on a $BT + AC$ structure $\mathfrak{S} = \langle Tree, \leq, Agent, Choice\rangle$ is a structure $\mathfrak{M} = \langle\langle Tree, \leq, Agent, Choice, \{\mathcal{B}^m_x \mid x \in Agent, m \in Tree,\}, \{\mathcal{D}^m_x \mid x \in Agent,\, m \in Tree\}, \{\mathcal{I}^m_x \mid x \in Agent,\, m \in Tree\}\rangle, \Im\rangle$, where $\langle \mathfrak{S}, \Im\rangle$ is a model and $\mathcal{B}_x^m \subseteq H_m \times H_m$, $\mathcal{D}_x^m \subseteq H_m \times H_m$, $\mathcal{I}_x^m \subseteq H_m \times H_m$. Thus, for every agent x and every moment m, we consider the set of *histories* passing through m that are compatible with what x believes (desires, intends) at a moment-history pair (m, h). We may assume that $\mathcal{B}_x^m$ is serial, transitive and Euclidean and that $\mathcal{D}_x^m$ and $\mathcal{I}_x^m$ are serial.

The new evaluation clauses are:

$$\mathfrak{M}, (m,h) \models \mathsf{BEL}_\alpha A \quad \text{iff} \quad (\forall h' \in H_m)\; h\,\mathcal{B}_\alpha^m\, h' \;\Rightarrow\; \mathfrak{M}, (m,h') \models A$$
$$\mathfrak{M}, (m,h) \models \mathsf{DES}_\alpha A \quad \text{iff} \quad (\forall h' \in H_m)\; h\,\mathcal{D}_\alpha^m\, h' \;\Rightarrow\; \mathfrak{M}, (m,h') \models A$$

$\mathfrak{M}, (m,h) \models \mathsf{INTEND}_\alpha[\alpha\; dstit\colon A]$ iff
$(\forall h' \in H_m)\; h\,\mathcal{I}_\alpha^m\, h' \;\Rightarrow\; ((\forall h'' \in Choice_m^{\mathfrak{I}(\alpha)}(h'))\; \mathfrak{M}, (m,h'') \models A$ and
$(\exists h''' \in H_m)\; \mathfrak{M}, (m,h''') \not\models A$

We take it that developing the combined logic of belief, desire, intention and dstit modalities with the suggested restricted complement condition is a promising avenue to future investigations.

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