Public Announcements and Belief Expansion

Hans van Ditmarsch, Wiebe van der Hoek and Barteld Kooi

ABSTRACT. In this paper we study the relation between two approaches to information change: Dynamic Epistemic Logic and Belief Revision. One of the main differences between these approaches is that higher-order information plays an important role in the field of Dynamic Epistemic Logic, whereas it does not feature in Belief Revision. In this paper we study to which extent public announcements (a particular kind of information change studied in Dynamic Epistemic Logic) can be viewed as a belief expansion (a particular kind of information change studied in Belief Revision).

1 Introduction

Since Hintikka's [9] epistemic logic, the logic of knowledge, has flourished as a research area in philosophy [10], computer science [6], artificial intelligence [12] and game theory [2]. The three mentioned application areas made it apparent that in multi-agent systems *higher-order information*, knowledge about (other) agents' knowledge, is crucial.

A natural question to ask, once the formal framework to reason about the information of agents — we will use the terms 'belief' and 'knowledge' interchangeably in this paper — is in place, is how the agent's belief changes over time, when he is confronted with new information; be it in a static, or an evolving world. The famous paper [1] by Alchourrón *et al.* put this change of information, coined as *Belief Revision (BR)*, as a topic on the philosophical and logical agenda: it was followed by a large stream of publications, on fine-tuning the notion of epistemic entrenchment [13], on revising (finite) belief bases [4], on differences between belief revision and belief updates [11], and the problem of iterated belief change [5].

This stream of research typically represents the beliefs of the (only) agent as a theory in propositional logic, and then describes, in terms of rationality postulates, how new information should be incorporated in it. Hence, in this approach the dynamics is studied on a level above the informational

Advances in Modal Logic, Volume 5.

level, not allowing reasoning about change of agents' knowledge and ignorance within the framework, let alone about the change of other agents' information from the perspective of a given agent.

Alternatively, the area of *Dynamic Epistemic Logic (DEL)* takes as its starting point modal epistemic logic in the tradition of [9], and adds a dynamic component to it. The aim here is not only to dynamize the epistemics, but also to 'epistemize the dynamics': the actions that (groups of) agents perform are epistemic actions. Different agents may have different information about which action is taking place, including higher-order information. This rather recent approach treats all of knowledge, higher-order knowledge, and its dynamics, on the same foot. Following an original contribution by Plaza in 1989 [14], a stream of publications appeared around the year 2000 [8, 3, 16, 17].

The relation between belief revision and dynamic modal logic is investigated by Segerberg in [15]. However, this study is limited to cases where higher-order information plays no role. In this paper we study a case with higher-order information, namely how a public announcement, a specific epistemic action in DEL, can be viewed as belief expansion, one of the three distinguished operations in BR. As such, our paper can be conceived as a next step in making the relation between the approaches explicit.

In Section 2 and 3 we briefly introduce public announcement logic and belief expansion, respectively. Section 4 demonstrates that belief expansion can be seen as a special case of public updates. In section 5 conclusions are drawn, and directions for further research are indicated.

2 Public announcement logic

Public announcement logic (PAL) was first developed by Plaza [14]. It extends epistemic logic to allow reasoning about information change due to public announcements: an epistemic action $[\varphi]$ where the whole group of agents are aware that they learn φ.

Definition 1 (Language of PAL). Let a finite set of propositional variables $\mathcal{P}$ and a finite set of agents $\mathcal{A}$ be given. The language $\mathscr{L}_{\mathsf{PAL}}$ is given by the following BNF:

$$\varphi ::= p \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \Box_a \varphi \mid [\varphi_1]\varphi_2$$

where $p \in \mathcal{P}$, $a \in \mathcal{A}$. Besides the usual abbreviations we use $E\varphi$ as an abbreviation for $\bigwedge_{a \in \mathcal{A}} \Box_a \varphi$.

A formula of the form $[\varphi]\psi$ has to be read as "ψ holds after the announcement of φ". This language is interpreted in models for epistemic logic. These are Kripke models with an accessibility relation for each agent.

All the logical languages presented in this paper are interpreted in these models.

Definition 2 (Epistemic models). Let a finite set of propositional variables $\mathcal{P}$ and a finite set of agents $\mathcal{A}$ be given. An epistemic model is a triple $M = (W, R, V)$ such that:

- $W \neq \emptyset$; a set of possible worlds;
- $R : \mathcal{A} \to 2^{W \times W}$; assigns an accessibility relation to each agent;
- $V : \mathcal{P} \to 2^{W}$; assigns a set of worlds to each propositional variable.

In epistemic logic, R is usually restricted to equivalence relations. In this paper we treat the weakest modal case where there are no restrictions on R, consequently most results also hold for stronger logics. The semantics are defined with respect to models with a distinguished world (the actual world): (M, w). We also call this a model.

The methodology used in dynamic epistemic logics is to regard epistemic actions as *state transformers*, i.e. they convey us from one model to another. The semantics are justified by showing how these state transformers work in examples, and by showing that the results agree with one's intuitions about these kinds of information change. The idea of public announcements is that all the agents simultaneously learn the announced formula, moreover it is common knowledge among them that they learn it. We assume that only *true* announcements can be made. This leads to the following semantics.

Definition 3 (Semantics of PAL). Let a model (M, w) with $M = (W, R, V)$ be given. Let $p \in \mathcal{P}$, $a \in \mathcal{A}$, and $\varphi, \psi \in \mathscr{L}_{\mathsf{PAL}}$.

$$
\begin{array}{lll}
(M,w) \models p & \text{iff} & w \in V(p) \\
(M,w) \models \neg\varphi & \text{iff} & (M,w) \not\models \varphi \\
(M,w) \models \varphi \wedge \psi & \text{iff} & (M,w) \models \varphi \text{ and } (M,w) \models \psi \\
(M,w) \models \Box_a \varphi & \text{iff} & (M,v) \models \varphi \text{ for all } v \text{ such that } (w,v) \in R(a) \\
(M,w) \models [\varphi]\psi & \text{iff} & (M,w) \models \varphi \text{ implies } (M|\varphi, w) \models \psi
\end{array}
$$

The model $M|\varphi = (W', R', V')$ is defined by restricting M to those worlds where φ holds. Let $[\![\varphi]\!] = \{v \in W | (M, v) \models \varphi\}$. Then $W' = [\![\varphi]\!]$, and the accessibility relations and valuation are restricted to $[\![\varphi]\!]$ as well: $R'(a) = R(a) \cap [\![\varphi]\!]^2$ and $V'(p) = V(p) \cap [\![\varphi]\!]$. If Γ is a set of formulas, then $Cn(\Gamma) = \{\varphi |$ for all (M, w), if $(M, w) \models \Gamma$, then $(M, w) \models \varphi\}$.

Definition 4 (Proof system for PAL). The proof system for PAL consists of all the axioms and rules of K (for modal operators $\Box_a$ and $[\varphi]$) plus the following axioms:

Atoms	$[\varphi]p \leftrightarrow (\varphi \rightarrow p)$	(atoms)
PF	$[\varphi]\neg\psi \leftrightarrow (\varphi \rightarrow \neg[\varphi]\psi)$	(partial functionality)
Distr	$[\varphi](\psi \wedge \chi) \leftrightarrow ([\varphi]\psi \wedge [\varphi]\chi)$	(distribution)
Ramsey	$[\varphi]\Box_a\psi \leftrightarrow (\varphi \rightarrow \Box_a[\varphi]\psi)$	(Ramsey axiom)

Completeness for this proof system is easy: One shows that every formula of the language of public announcements can be translated to a provably equivalent formula without announcement operators.

Definition 5 (Translation). The translation function t takes a formula from the language of PAL and yields a formula in the language of epistemic logic, in such a way that $t(p) = p$, and t distributes over $\neg, \wedge$ and $\Box_a$. The announcement operator is treated as follows:

$$\begin{array}{lcl} t([\varphi]p) & = & t(\varphi) \rightarrow p \\ t([\varphi]\neg\psi) & = & t(\varphi) \rightarrow \neg t([\varphi]\psi) \\ t([\varphi](\psi \wedge \chi)) & = & t([\varphi]\psi) \wedge t([\varphi]\chi) \\ t([\varphi]\Box_a\psi) & = & t(\varphi) \rightarrow \Box_a t([\varphi]\psi) \\ t([\varphi][\psi]\chi) & = & t([\varphi]t([\psi]\chi)) \end{array}$$

Lemma 1 (Plaza). $\vdash \varphi \leftrightarrow t(\varphi)$ for every formula φ.

Some have commented on the fact that every formula containing announcements is equivalent to a formula without any announcements, by remarking that announcements are merely syntactic sugar. However, applying the same line of argument to the study of propositional logic, one would be forced to say that one can make do with a language containing Sheffer's stroke as the only logical operator. Although this is technically feasible, it makes no sense philosophically, and practically it is quite bothersome. And, in the case of public announcement logic the above translation also demonstrates its succintness with respect to epistemic logic, since the translation may give a formula with a size exponentional in its input.

The next definition and lemma will be used in section 4.

Definition 6 (Epistemic depth). The epistemic depth of a formula is a function $d : \mathscr{L}_{\mathsf{PAL}} \rightarrow \mathbb{N}$ such that:

$$\begin{array}{lcl} d(p) & = & 0 \\ d(\neg\varphi) & = & d(\varphi) \\ d(\varphi \wedge \psi) & = & \max(d(\varphi), d(\psi)) \\ d(\Box_a\varphi) & = & d(\varphi) + 1 \\ d([\varphi]\psi)) & = & d(\varphi) + d(\psi) \end{array}$$

Lemma 2. For all formula $\varphi \in \mathscr{L}_{\mathsf{PAL}}$, φ and $t(\varphi)$ have the same epistemic depth.

Proof. By strong induction on n. The base case is easy. For the induction step one can simply check every clause of the translation. ■

Example 1. For an example, consider the formula $[\Box_a p]\Box_b q$. Its translation into epistemic logic is $t([\Box_a p]\Box_b q) = t(\Box_a p) \to \Box_b t([\Box_a p]q) = \Box_a p \to \Box_b(t(\Box_a p) \to q) = \Box_a p \to \Box_b(\Box_a p \to q)$. According to the definition of modal depth, $d([\Box_a p]\Box_b q) = 2$ and this indeed reflects that its translation $\Box_a p \to \Box_b(\Box_a p \to q)$ has a stack of two epistemic operators.

The following lemma will be used in the the proof of Theorem 3 in Section 4.

Lemma 3. Let $d(\psi) = n$. Then: $\vdash E^n\varphi \to ([\varphi]\psi \leftrightarrow \psi)$

Proof. If $d(\psi) = 0$, it follows straightforwardly from atoms, partial functionality and distribution.

If $d(\psi) = n + 1$, the interesting case is when ψ is of the form $\Box_a\chi$. Then by the Ramsey axiom $\vdash [\varphi]\Box_a\chi \leftrightarrow (\varphi \to \Box_a[\varphi]\chi)$. By propositional reasoning we get $\vdash (E^{n+1}\varphi \to [\varphi]\Box_a\chi) \leftrightarrow (E^{n+1}\varphi \to (\varphi \to \Box_a[\varphi]\psi))$. Observe that $\vdash E^{n+1}\varphi \to \varphi$ and $\vdash E^{n+1}\varphi \to \Box_a E^n\varphi$. With some modal reasoning we get $\vdash (E^{n+1}\varphi \to [\varphi]\Box_a\chi) \leftrightarrow (E^{n+1}\varphi \to \Box_a(E^n\varphi \wedge [\varphi]\psi)$. The induction hypothesis tells us that $\vdash E^n\varphi \to ([\varphi]\psi \leftrightarrow \psi)$. Therefore $\vdash (E^{n+1}\varphi \to [\varphi]\Box_a\chi) \leftrightarrow (E^{n+1}\varphi \to \Box_a(E^n\varphi \wedge \psi))$. With some more modal and propositional reasoning we finally get $E^n\varphi \to ([\varphi]\Box_a\chi \leftrightarrow \Box_a\chi)$. ■

3 Belief expansion

In belief revision the beliefs of an agent are not represented by means of a Kripke model, but with a set of propositional formulas, called a belief set. Therefore all the logical notions refer to propositional logic.

Definition 7 (Belief sets). A belief set K is a set of formulas closed under logical consequence, i.e. $K = Cn(K)$.

One of the types of information change studied in belief revision is *belief expansion*. When an agent accepts the information that φ, the agent is said to *expand* his or her beliefs with φ. The methodology used in this branch of logic, is to state properties that such a change of information satisfies, assuming that the agent performing the information change is rational. These are called *rationality postulates*, which can be justified by arguments that appeal to one's intuition about the information change. Then one can set out to prove that these postulates uniquely characterize a certain operation on belief sets. In [1] the expansion operation $+$ is characterized by the following rationality postulates.

Definition 8 (Rationality postulates for belief expansion).

K1 $K + \varphi$ is a belief set.

K2 $\varphi \in K + \varphi$.

K3 $K \subseteq K + \varphi$.

K4 If $\varphi \in K$, then $K + \varphi = K$.

K5 If $K \subseteq K'$, then $K + \varphi \subseteq K' + \varphi$.

K6 $K + \varphi$ is the smallest belief set that satisfies **K1-K5**.

This operation turns out to be fully characterized by these six postulates. This leads to the following theorem.

Theorem 1 (Gärdenfors [7]). The expansion function $+$ satisfies **K1-K6** iff $K + \varphi = Cn(K \cup \{\varphi\})$

So the operation of expansion is simply adding the new information to the agent's belief set and closing under logical consequence. In the next section we show that under certain conditions, public announcements can also be seen as expansions.

4 Positive knowledge

So far, we have seen two logical approaches that provide an analysis of the same kind of information change: incorporating new information. Therefore one would expect that, although technically they are worked out differently, there are great similarities. However, the similarities turn out to be limited to a very special case. The postulates of expansion, discussed in Section 3, do not straightforwardly translate to properties in the context of dynamic epistemic logic. First of all, many logics for knowledge and belief assume the properties of *positive* ($\Box\varphi \rightarrow \Box\Box\varphi$) and *negative introspection* ($\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$) and this makes it very hard to compare belief sets, because these cannot even be expressed in terms of belief sets of propositional formulas. But even if we extend belief sets with modal formulas there is a problem. Suppose we take a belief set to be some set $K_{(M,w)} = \{\varphi \mid (M,w) \models \Box\varphi\}$, for a given model (M,w). Take two such sets K and K'. Now suppose that $K \subseteq K'$. This implies that $K = K'$. This can be seen as follows. Suppose it is not the case that $K' \subseteq K$. Then there is a formula $\psi \in K'$

and $\psi \notin K$. If $\psi \in K'$, then by positive introspection, it follows that $\Box\psi \in K'$. On the other hand, $\psi \notin K$ implies that in the underlying model $(M, w) \not\models \Box\psi$. Therefore by negative introspection $(M, w) \models \Box\neg\Box\psi$, and so $\neg\Box\psi \in K$. This together with the assumption that $K \subseteq K'$ also gives $\neg\Box\psi \in K'$. And so we have arrived at a contradiction. In other words, fully introspective agents can never add or remove beliefs, without making their belief set inconsistent!

Secondly, postulate **K2** of expansions ($\varphi \in K + \varphi$, also called *success*), is hard to enforce in a setting where we allow for epistemic operators in the object language. To see this, for any epistemic logic satisfying the weak variant $\Box\neg\Box\varphi \rightarrow \neg\Box\varphi$ of veridicality, it not easy to see how an expansion of K with, say, $p \wedge \neg\Box p$ should look like. By the postulate of success, we have $(p \wedge \neg\Box p) \in K$, which is impossible to fulfill in combination with our weak notion of veridicality and the requirements $K \not\vdash \bot$ and $K = Cn(K)$.

In this section we show that public announcements *do* behave like belief expansions, when we restrict ourselves to *positive knowledge*, and when the formulas that are learnt are *successful*. The problem (that beliefs sets are incomparable) and the analysis below is not unlike the one presented in [18], where the driving question is what it means that an agent claims to 'only know' some fact φ.

For this purpose we look explicitly at the levels of knowledge that an agent possesses. This leads to the following inductive definition of the positive fragment of the language of public announcement logic.

Definition 9 ($\mathscr{L}_{pos}$). The languages $\mathscr{L}^n_{pos}$ (for each $n \in \mathbb{N}$) are defined inductively as follows, where k is any value smaller than or equal to $n+1$:

$$\begin{array}{lll} \varphi_0 & ::= & p \mid \neg p \mid \varphi_0 \vee \varphi_0 \mid \varphi_0 \wedge \varphi_0 \\ \varphi_{n+1} & ::= & \varphi_n \mid \Box_a \varphi_n \mid [\neg\varphi_k]\varphi_{(n+1)-k} \mid \varphi_{n+1} \vee \varphi_{n+1} \mid \varphi_{n+1} \wedge \varphi_{n+1} \end{array}$$

$\mathscr{L}_{pos} = \bigcup_{n \in \mathbb{N}} \mathscr{L}^n_{pos}$.

Note that if $\varphi \in \mathscr{L}^n_{pos}$, then $d(\varphi) \leq n$. Also note that if $\varphi \in \mathscr{L}^n_{pos}$, then there is a formula ψ without announcement operators, which is equivalent to φ, such that $\psi \in \mathscr{L}^n_{pos}$, by lemma's 1 and 2. Therefore, in the following proofs by induction on $\mathscr{L}^n_{pos}$ we omit the case for $[\varphi]\psi$.

Definition 10 (Finite simulation). Let two models $M = (W, R, V)$ and $M' = (W', R', V')$, and two worlds $w \in W$ and $w' \in W'$ be given. $S \subseteq W \times W'$ is a simulation up to 0 for w and w' iff wSw and for all $p \in \mathcal{P}$ it is the case that $w \in V(p)$ iff $w' \in V'(p)$. $S \subseteq W \times W'$ is a simulation up to $n+1$ for w and w' iff

atoms S is a simulation up to 0 for w and w', and

forth for every $a \in \mathcal{A}$ and $v \in W$, if $wR(a)v$, then there is a $v' \in W'$ such that $w'R'(a)v'$ and S is a simulation up to n for v and v'.

If there is a simulation up to n for w and w' we write $(M, w) \rightrightarrows_n (M', w')$. If there is a simulation up to every $n \in \mathbb{N}$ for we write $(M, w) \rightrightarrows_\omega (M', w')$

The following lemma holds due to the fact that we only have a finite number of propositional variables and only a finite number of agents, and because we only look at formulas up to a certain depth.

Lemma 4 (Propositional finiteness). The number of different propositions that can be expressed by formulas of $\mathscr{L}^n_{pos}$ is finite for every n. More formally: logical equivalence yields a finite partition on $\mathscr{L}^n_{pos}$.

So although there are infinitely many different formulas in the language up to n, only finitely many things can be expressed with it.

Definition 11. Let $Th^n_{pos}(M, w) = \{\varphi \in \mathscr{L}^n_{pos} | (M, w) \models \varphi\}$

Theorem 2. $(M, w) \rightrightarrows_n (M', w')$ iff $Th^n_{pos}(M', w') \subseteq Th^n_{pos}(M, w)$

Proof. By induction on n, the base case being trivial. So suppose the theorem holds up to n.

($\Rightarrow$) Suppose $(M, w) \rightrightarrows_{n+1} (M', w')$. Let $\varphi \in \mathscr{L}^{n+1}_{pos}$. We proceed by induction on φ. If $\varphi \in \mathscr{L}^n_{pos}$ we can simply apply the induction hypothesis. Suppose φ is of the form $\Box_a \psi$. By contraposition. Suppose $(M, w) \not\models \Box_a \psi$. Then there is a world v such that $wR(a)v$ and $(M, v) \not\models \psi$. From forth it follows that there is a v' such that $w'R(a)v'$ and $(M, v) \rightrightarrows_n (M', v')$. Since $\psi \in \mathscr{L}^n_{pos}$, we can apply the contrapositive of the induction hypothesis. Therefore $(M', v') \not\models \psi$, and therefore $(M', w') \not\models \Box_a \psi$.

($\Leftarrow$) Suppose that $Th^{n+1}_{pos}(M', w') \subseteq Th^{n+1}_{pos}(M, w)$. Now let S be, for all worlds: $S = \{(v, v') | Th^{n+1}_{pos}(M', v') \subseteq Th^{n+1}_{pos}(M, v)\}$. We prove that S is a simulation up to $n+1$. The **atoms** clause follows directly. Suppose toward a contradiction that there is a $wR(a)v$ and for all $v' \in W'$ such that $w'R'(a)v'$ the relation S is not a simulation up to n for v and v'. By the induction hypothesis there is a formula $\psi_{vv'} \in \mathscr{L}^n_{pos}$ such that $(M', v') \models \psi_{vv'}$ and $(M, v) \not\models \psi_{vv'}$. Let $\Gamma = \{\psi_{vv'} | w'R'(a)v'\}$. We can assume this set is finite, without loss of generality, by Lemma 4. Therefore $(M', w') \models \Box_a \bigvee \Gamma$ and $(M, w) \not\models \Box_a \bigvee \Gamma$. But this formula is in $\mathscr{L}^{n+1}_{pos}$, therefore $(M, w) \models \Box_a \bigvee \Gamma$. And so we arrive at a contradiction. ∎

Example 2. The supermodel relation yields a simulation up to an arbitrary depth. This also shows that the submodel relationship is *not* a simulation beyond a certain level.

For example, consider the following four-state epistemic state (M, w): the set of possible worlds of the model M is $\{w, w', v, v'\}$; there is one fact p,

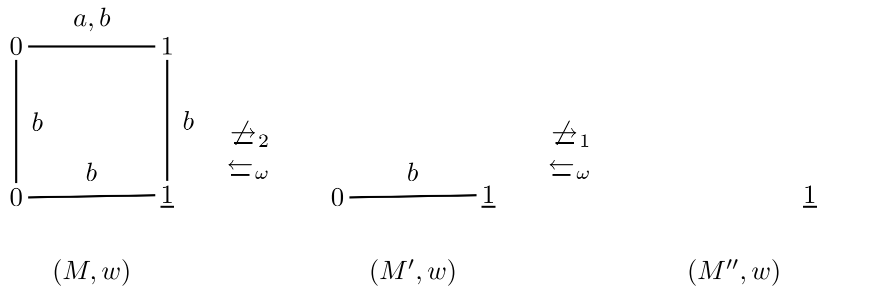

Figure 1. Simulation relations between three example models. All are defined for one atom p only, and two agents a and b. The point of the model is underlined. In states named 0 fact p is false, and in states named 1 true. All access is symmetric and transitive, and reflexive access is assumed as well. For example, in the leftmost model access for b is the universal relation; and in the rightmost model access for both a and b is reflexive. The last depicts the situation where they both know that p (and where they know from each other that they know that, etc.).

which is true in w, w' only, access for agent b is universal, and access for a induces the partition $\{w, w'\}, \{v\}, \{v'\}$. The model represents an epistemic state where agent b suspects agent a of knowing the truth about p. We further consider the submodels (M', w) and (M'', w): M' is the submodel of M for restricted domain $\{v, w\}$, and M'' is the restriction of M to $\{w\}$ only. These models are visualized in Figure 1, including their relevant simulation relations. We now observe the following, which is a direct application of Theorem 2.

The formula $\Box_a p$ is true in (M, w). Therefore, by Theorem 2, and because of $(M'', w) \rightleftarrows_\omega (M', w) \rightleftarrows_\omega (M, w)$, it is also true in (M', w) and (M'', w). The formula $\Box_b \neg p$ is false in (M'', w). Therefore, because of $(M'', w) \rightleftarrows_\omega (M', w) \rightleftarrows_\omega (M, w)$, it is also false in (M', w) and (M, w). To show that there is no simulation for (M, w) and (M', w') up to 2, it suffices to observe that $\Box_b(\Box_a p \vee \Box_a \neg p) \in Th^2_{pos}(M', w)$ but $\Box_b(\Box_a p \vee \Box_a \neg p) \notin Th^2_{pos}(M, w)$. There is no simulation for (M', w) and (M'', w) up to 1, because $\Box_b p \in Th^1_{pos}(M'', w)$ but $\Box_b p \notin Th^1_{pos}(M', w)$.

Let $Cn^n_{pos}(\Gamma) = \{\varphi \in \mathscr{L}^n_{pos} |$ for all (M, w), if $(M, w) \models \Gamma$, then $(M, w) \models \varphi\}$.

Theorem 3. Let $\varphi \in \mathscr{L}^n_{pos}$, and let $\neg\varphi$ be successful, i.e. $\models [\neg\varphi]\neg\varphi$, then

$$Th^n_{pos}(M|\neg\varphi, w) = Cn^n_{pos}(Th^n_{pos}(M, w) \cup \{E^n \neg\varphi\})$$

Proof. ($\Rightarrow$) Let $\psi \in \mathscr{L}^n_{pos}$. Suppose $\psi \in Th^n_{pos}(M|\neg\varphi, w)$, i.e. $(M|\neg\varphi, w) \models \psi$. Let (M', w') be an arbitrary model such that $(M', w') \models Th^n_{pos}(M, w) \cup \{E^n\neg\varphi\}$. Therefore $Th^n_{pos}(M, w) \subseteq Th^n_{pos}(M', w')$. By Theorem 2, this is equivalent to $(M', w') \rightrightarrows_n (M, w)$. Therefore $(M'|\neg\varphi, w') \rightrightarrows_n (M|\neg\varphi, w)$. Since $\psi \in \mathscr{L}^n_{pos}$, by Theorem 2, $(M'|\neg\varphi, w') \models \psi$. Therefore $(M', w') \models [\neg\varphi]\psi$. Since $(M', w') \models E^n\neg\varphi$, by Lemma 3, $(M', w') \models \psi$.

($\Leftarrow$) It is sufficient to show that $(M|\neg\varphi, w) \models Th^n_{pos}(M, w) \cup \{E^n\neg\varphi\}$. It is easy to see that $(M|\neg\varphi, w) \rightrightarrows_n (M, w)$. Therefore by Lemma 3 $(M|\neg\varphi, w) \models Th^n_{pos}(M, w)$. Suppose that $(M|\neg\varphi, w) \not\models E^n\neg\varphi$. Then there is a world v accessible from w with a path of length at most n such that $(M|\neg\varphi, v) \models \varphi$. This contradicts $\models [\neg\varphi]\neg\varphi$. Therefore $(M|\neg\varphi, w) \models E^n\neg\varphi$. ■

This theorem shows that those positive formulas which are true after a successful announcement of a negation of a positive formula φ, are exactly those positive formulas which follow from the positive knowledge before the announcement together with knowledge of $\neg\varphi$ up to a certain depth. In order to make the link with belief expansion more clear, we state the following two corollaries, which look at the positive knowledge of one agent.

Corollary 1. Let $K^n_{(M,w)}(a) = \{\varphi \in \mathscr{L}^n_{pos} | (M, w) \models \Box_a\varphi\}$. Let $\varphi \in \mathscr{L}^n_{pos}$, and let $\neg\varphi$ be successful, then $K^n_{(M|\neg\varphi,w)}(a) = Cn^n_{pos}(K^n_{(M,w)}(a) \cup \{E^n\neg\varphi\})$

Propositional formulas are always successful. Therefore we do not need it as an explicit assumption for $\mathscr{L}^0_{pos}$. Also, we can use the ordinary notion of consequence. This leads to the following corollary, saying that public announcements are the same as expansions on a propositional level.

Corollary 2. For $\mathscr{L}^0_{pos}$, $K^0_{(M|\neg\varphi,w)}(a) = Cn(K^0_{(M,w)}(a) \cup \{\neg\varphi\})$

5 Conclusion and further research

In this paper we investigated to what extent public announcement and belief expansion are the same. We showed that the two notions coincide as far as positive knowledge is concerned, but it seems difficult to extend this to broader sublanguages of public announcement logic. This indicates, that even in the simple case of learning something new, one's intuitions may fail. It also poses the question which methodology is to be preferred when studying information change.

There are three obvious main directions for further research on the short term. First of all, now that we have determined a counterpart of *expansions* in DEL, a natural next enterprise is to extend DEL to a framework in which one can characterize a notion of *contraction*, and, once that is in place, a notion of *revision*. The other two directions are partly suggested by similar problems when studying 'only knowing' [18]. First, it is not entirely clear at this stage how our results generalize to, or are dependent, on specific axiomatic systems. Finally, we should expand our analysis to a real *multi-agent* framework, allowing higher order belief revision for individual agents in a larger group, unlike the *public* expansion we have modelled her. Where in our current approach the agent's knowledge can not occur in the scope of a negation, it is not clear how this generalizes to the case of more agents: maybe a formula of type $\Box_a \neg \Box_b \Box_a p$ would not disturb the results proven here.

Acknowledgements

Hans van Ditmarsch thanks the Netherlands Organisation for Scientific Research (NWO) for a grant to facilitate collaboration with Barteld Kooi.

BIBLIOGRAPHY

[1] C. E. Alchourrón, P. Gärdenfors, and D. Makinson. On the logic of theory change: partial meet functions for contraction and revision. *Journal of Symbolic Logic*, 50:510–530, 1985.

[2] R. J. Aumann and A. Brandenburger. Epistemic conditions for Nash equilibrium. *Econometrica*, 63(5):1161–1180, 1995.

[3] A. Baltag and L.S. Moss. Logics for epistemic programs. *Synthese*, 139:165–224, 2004. Knowledge, Rationality & Action 1–60.

[4] S. Benferhat, D. Dubois, H. Prade, and M.A. Williams. A practical approach to revising prioritized knowledge bases. *Studia Logica*, 70(1), 2002.

[5] A. Darwiche and J. Pearl. On the logic of iterated belief revision. *Artificial Intelligence*, 89(1-2):1–29, 1997.

[6] R. Fagin, J.Y. Halpern, Y. Moses, and M.Y. Vardi. *Reasoning About Knowledge*. MIT Press, Cambridge, Massachusetts, 1995.

[7] P. Gärdenfors. *Knowledge in Flux, modeling the dynamics of epistemic states*. MIT Press, 1988.

[8] J. Gerbrandy and W. Groeneveld. Reasoning about information change. *Journal of Logic, Language, and Information*, 6:147 –196, 1997.

[9] J. Hintikka. *Knowledge and Belief, An Introduction to the Logic of the Two Notions*. Cornell University Press, Ithaca & London, 1962.

[10] J. Hintikka. Reasoning about knowledge in philosophy. In J. Y. Halpern, editor, *Proceedings of Theoretical Aspects of Reasoning About Knowledge*, pages 63–80. Morgan Kaufmann Publishers, 1986.

[11] H. Katsuno and A. Mendelzon. On the difference between updating a knowledge base and revising it. In *Proceedings of the Second International Conference on Principles of Knowledge Representation and Reasoning*, pages 387–394, 1991.

[12] J.-J.Ch. Meyer and W. van der Hoek. *Epistemic Logic for AI and Computer Science*. Cambridge University Press, Cambridge, 1995.

[13] T.A. Meyer, W.A. Labuschagne, and J. Heidema. Refined epistemic entrenchment. *Journal of Logic, Language and Information*, 9:237–259, 2000.

[14] J. A. Plaza. Logics of public communications. In M. L. Emrich, M. S. Pfeifer, M. Hadzikadic, and Z. W. Ras, editors, *Proceedings of the 4th International Symposium on Methodologies for Intelligent Systems*, pages 201–216, 1989.

[15] K. Segerberg. Two traditions in the logic of belief: bringing them together. In H. J. Ohlbach and U. Reyle, editors, *Logic, Language and Reasoning: essays in honour of Dov Gabbay*, volume 5 of *Trends in Logic*, pages 135–147. Kluwer Academic Publishers, Dordrecht, 1999.

[16] H. P. van Ditmarsch. Descriptions of game actions. *Journal of Logic, Language and Information*, 11(3):349–365, 2002.

[17] H.P. van Ditmarsch, W. van der Hoek, and B.P. Kooi. Concurrent dynamic epistemic logic. In V.F. Hendricks, K.F. Jørgensen, and S.A. Pedersen, editors, *Knowledge Contributors*, pages 45–82. Kluwer Academic Publishers, Dordrecht, 2003. Synthese Library Volume 322.

[18] J. Jaspars W. van der Hoek and E. Thijsse. Persistence and minimality in epistemic logic. *Annals of Mathematics and Artificial Intelligence*, 27(1–4):25–47, 1999 (printed 2000).

Hans van Ditmarsch

Department of Computer Science, University of Otago, PO Box 56, Dunedin 9015, New Zealand,

hans@cs.otago.ac.nz

Wiebe van der Hoek

Department of Computer Science, University of Liverpool, Liverpool L69 7ZF, United
Kingdom

wiebe@csc.liv.ac.uk

Barteld Kooi

Department of Philosophy, University of Groningen, Oude Boteringestraat 52, 9712 GL Groningen, The Netherlands

barteld@philos.rug.nl