

# Non-classical modal logic for natural language

Wesley H. Holliday

*University of California, Berkeley*

---

## Abstract

Modality is a central topic in natural language semantics [18,27,26,19,20]. Modal logic has contributed to the study of modality in natural language in important ways, including by providing perspicuous axiomatizations of the consequence relations defined by proposed formal semantics for fragments of modal language (see [13] for an overview). In this talk, based on joint work with Matthew Mandelkern [14], I will explain how attempting to solve one of the most discussed problems in modal semantics in recent years—accounting for the peculiar empirical behavior of *epistemic* modals [28,8,29,1,6,30,21,17,5,4,24,25,23,9,16,2]—led us to a non-classical modal logic for the relevant fragment of language. We have characterized the logic axiomatically as well as semantically, using algebraic semantics based on ortholattices, relational representations of ortholattices [3,22,7], and possibility semantics for modals [15,10,12,11]. Thus, I will discuss how the kinds of tools developed in the AiML community for mathematical modal logic can be brought to bear on a well-known applied problem.

---

## References

- [1] Aloni, M., *Conceptual covers in dynamic semantics*, in: L. Cavedon, P. Blackburn, N. Braisby and A. Shimojima, editors, *Logic, Language and Computation, Vol. III*, CSLI, 2000 .
- [2] Aloni, M., L. Incurvati and J. J. Schöder, *Epistemic modals in hypothetical reasoning*, *Erkenntnis* (Forthcoming).
- [3] Birkhoff, G., “Lattice Theory,” American Mathematical Society, New York, 1940.
- [4] Bledin, J., *Logic informed*, *Mind* **123** (2014), pp. 277–316.
- [5] Dorr, C. and J. Hawthorne, *Embedding epistemic modals*, *Mind* **122** (2013), pp. 867–913.
- [6] Gillies, A., *Epistemic conditionals and conditional epistemics*, *Noûs* **38** (2004), pp. 585–616.
- [7] Goldblatt, R. I., *Semantic analysis of orthologic*, *Journal of Philosophical Logic* **3** (1974), pp. 19–35.
- [8] Groenendijk, J., M. Stokhof and F. Veltman, *Coreference and modality*, in: S. Lappin, editor, *Handbook of Contemporary Semantic Theory*, Oxford, Blackwell, 1996 pp. 179–216.
- [9] Hawke, P. and S. Steinert-Threlkeld, *Semantic expressivism for epistemic modals*, *Linguistics and Philosophy* **44** (2021), pp. 475–511.
- [10] Holliday, W. H., *Partiality and adjointness in modal logic*, in: R. Goré, B. Kooi and A. Kurucz, editors, *Advances in Modal Logic, Vol. 10*, College Publications, London, 2014 pp. 313–332.

- [11] Holliday, W. H., *Possibility semantics*, in: M. Fitting, editor, *Selected Topics from Contemporary Logics*, Landscapes in Logic, College Publications, London, 2021 pp. 363–476.
- [12] Holliday, W. H., *Possibility frames and forcing for modal logic*, The Australasian Journal of Logic (Forthcoming).  
URL <https://escholarship.org/uc/item/0tm6b30q>
- [13] Holliday, W. H. and T. F. Icard, *Axiomatization in the meaning sciences*, in: D. Ball and B. Rabern, editors, *The Science of Meaning: Essays on the Metatheory of Natural Language Semantics*, Oxford University Press, 2018 pp. 73–97.
- [14] Holliday, W. H. and M. Mandelkern, *The orthologic of epistemic modals* (2021), arXiv:2203.02872 [cs.LO].
- [15] Humberstone, I. L., *From worlds to possibilities*, Journal of Philosophical Logic **10** (1981), pp. 313–339.
- [16] Incurvati, L. and J. J. Schlöder, *Epistemic multilateral logic*, Review of Symbolic Logic **15** (2022), pp. 505–536.
- [17] Klinedinst, N. and D. Rothschild, *Connectives without truth-tables*, Natural Language Semantics **20** (2012), pp. 137–175.
- [18] Kratzer, A., *Modality*, in: A. von Stechow and D. Wunderlich, editors, *Semantics: An International Handbook of Contemporary Research*, de Gruyter, Berlin, 1991 pp. 639–650.
- [19] Kratzer, A., “Modals and Conditionals,” Oxford University Press, 2012.
- [20] Lassiter, D., “Graded Modality,” Oxford University Press, 2017.
- [21] MacFarlane, J., *Epistemic modals are assessment sensitive*, in: A. Egan and B. Weatherson, editors, *Epistemic Modality*, Oxford University Press, 2011 pp. 144–177.
- [22] MacLaren, M. D., *Atomic orthocomplemented lattices*, Pacific Journal of Mathematics **14** (1964), pp. 597–612.
- [23] Mandelkern, M., *Bounded modality*, The Philosophical Review **128** (2019), pp. 1–61.
- [24] Moss, S., *On the semantics and pragmatics of epistemic vocabulary*, Semantics and Pragmatics **8** (2015), pp. 1–81.
- [25] Ninan, D., *Quantification and epistemic modality*, The Philosophical Review **127** (2018), pp. 433–485.
- [26] Portner, P., “Modality,” Oxford University Press, 2009.
- [27] Swanson, E., *Modality in language*, Philosophy Compass **3** (2008), pp. 1193–1207.
- [28] Veltman, F., “Logics for Conditionals,” Ph.D. thesis, University of Amsterdam (1985).
- [29] Veltman, F., *Defaults in update semantics*, Journal of Philosophical Logic **25** (1996), pp. 221–261.
- [30] Yalcin, S., *Epistemic modals*, Mind **116** (2007), pp. 983–1026.