

# About ‘Binding Modalities’

Melvin Fitting<sup>1</sup>

*Graduate Center, City University of New York*  
*web page: melvinfitting.org*

In classical logic the addition of quantifiers to propositional logic is essentially unique, with some minor variations of course. In modal logic things are not so monolithic. One can quantify over things or over intensions; domains can be the same from possible world to possible world, or shrink, or grow, or follow no pattern, as one moves from a possible world to an accessible one. In 1963 Kripke showed that shrinking or growing domains related to validity of the Barcan and the converse Barcan formulas, but this was a semantic result. Proof theory is trickier. Nested sequents are well behaved, but axiom systems can be unruly. A direct combination of propositional modal axioms and rules with standard quantificational axioms and rules simply proves the converse Barcan formula. It’s not easy to get rid of it. Kripke showed how one could do so, but he needed to use a less common axiomatization of the quantifiers. It works, but one has the impression of having a formal proof system with road blocks placed carefully to prevent proofs from veering into the ditch.

Some 40 or more years later, justification logic was created by Artemov, and now there are justification systems that correspond to infinitely many different modal logics. The first justification logic was called LP, for “logic of proofs”. It is related to propositional S4. LP was extended to a quantified version by Artemov and Yavorskaya, with a possible world semantics supplied by Fitting. Subsequently Artemov and Yavorskaya transferred their ideas, concerning what they called “binding modalities”, back from quantified LP to quantified S4 itself. In the present work we carry their ideas on further to the basic normal modal logic, K, which is not as well-behaved as S4 on these matters. It turns out that this provides a natural intuition for Kripke’s non-standard axiomatization from those many years ago. It also relates quite plausibly to the distinction between *de re* and *de dicto*. But now the main work is done through a generalization of the modal operator, instead of through a restriction on allowed quantifier axiomatizations.

---

<sup>1</sup> melvin.fitting@gmail.com