An interval logic for natural language semantics

Savas Konur

ABSTRACT. Most temporal logics, particularly interval temporal logics, are not expressive enough to capture meanings of natural language constructions, and they are not convenient to represent temporal expressions. In addition, these formal systems exhibit high computational complexity. In this paper we introduce a decidable *event-based interval logic*, called *EIL*. EIL can express the semantics of some natural language constructions.

Keywords: interval temporal logics, natural language semantics, temporal prepositions, decidability, complexity, tableau-methods

1 Introduction

In a sentence of natural language temporal information is stored in temporal constructions such as prepositions. In order to understand the semantics of a sentence in English or in any other language it is very important to capture temporal meanings. Sentences encoding temporal information usually speak of events and their temporal relations. A natural question that arises here is what is the computational complexity of determining logical relationships between sentences encoding temporal information?

This question is of theoretical interest, because events in sentences with temporal information are extended in time; and temporal logics which deal with extended events so-called interval temporal logics, typically exhibit high computational complexity. Generally speaking, these logics are not expressive enough to capture the meanings of natural language constructions, and therefore they are not convenient to represent temporal expressions.

The formal semantics of temporal constructions in English have been studied by various researchers [4, 7, 1, 6, 11, 10, 8]. In most of the cases the issues related to computational complexity and expressive power are rarely investigated. In fact, in many cases, the semantics of temporal constructions in a natural language are represented in a first-order language having variables which range over time-intervals and predicates which correspond to event-types and temporal order-relations. Such a logic can be easily shown to be undecidable.

The recent interest is using logical fragments of limited computational complexity, because there are evident practical and theoretical reasons for presenting the semantics of natural language constructions, using formal systems of limited expressive power. In particular, this is important to achieve the decidability of such systems.

In this paper, we introduce a decidable *event-based interval temporal logic*, called *EIL*. The logic EIL has a limited expressive power; yet it has affinity with the syntax of temporal constructions in English, and it is convenient for expressing the semantics of natural language constructions.

In the literature various methods have been proposed to achieve decidability for interval logics. However, most of the methods, such as translating interval logics into point-based ones, cause some syntactic and semantic restrictions. A major challenge in this area is thus to *genuinely* identify interval-based decidable logics, that is, logics which are not explicitly translated into point-based logics or other semantic restrictions.

Unlike many other interval logics, we do not translate the logic EIL into a point-based variant, and we therefore try to minimise semantic restrictions. Instead, we consider intervals as primitive objects of the model by allowing quantification over only interval objects. Another important feature of EIL is that it incorporates the notion of *duration*, which denotes the length of the time period at which an event occurs (that is, it starts and finishes).

In this paper we propose a tableau-based decision procedure for EIL, thus showing that its satisfiability problem is decidable. We, indeed, provide a complexity bound for satisfiability, showing that this problem can be solved in 2-NEXPTIME. The tableau method we introduce decides whether the given formula is satisfiable or not, and generates a model if the formula is satisfiable.

The plan of this paper is as follows. In Section 2 the syntax and semantics of EIL will be presented. In Section 3 we will give a depth bound for EIL models. In Section 4 a tableau system will be proposed for the logic. In Section 5 we will give some concluding remarks, and discuss some future work.

2 The logic EIL

In this section we present syntax and semantics of the logic EIL. In the rest of this paper we take an *interval* to be a closed, bounded and non-empty subset of the real line. More formally we say that an *interval* is a pair $[t_1, t_2]$ such that $t_1, t_2 \in \mathbb{R}$ and $t_1 \leq t_2$. We denote the set of all intervals $\{[t_1, t_2] : t_1 \leq t_2 \wedge t_1, t_2 \in \mathbb{R}\}$ by $\mathcal{I}$, and we use letters $I, J, ...,$ as intervals. It can be simply observed that intervals may be points. Note also the temporal domain is continuous. EIL formulas are evaluated relative to time-intervals. As will be seen later, having event-types in the syntax of the language will allow us to formalize event-based sentences of a natural language. EIL also incorporates the notion of duration (of an event).

Event types are denoted by the letters $e_1, e_2, e_3, ...$ We interpret an event e so that it is satisfied by all and only those time intervals over which e occurs. We will think of $\langle e \rangle$ as the occurrence of e over an interval J. Below we define some functions on $\mathcal{I}$.

DEFINITION 1. Let $J, I \in \mathcal{I}$ be the intervals $[a,b]$ and $[c,d]$, respectively, with $a \leq c \leq d \leq b$. The terms $init(J,I)$ and $fin(J,I)$ denotes the intervals $[a,c]$ and $[d,b]$, respectively, where $init$ and fin are partial functions.

In the sequel, let $\mathcal{E}$ be a finite set. We refer to elements of $\mathcal{E}$ as *event atoms.*

DEFINITION 2. Let $e \in \mathcal{E}$ be an event atom, ϕ, ψ be EIL formulas, $k \in \mathbb{R}$, and $\tau \in \{<, \leq, =, \geq, >\}$. The logic EIL is defined by induction as follows:

$$\begin{aligned}\phi ::= \top \mid \left\langle \textstyle\int e\tau k \right\rangle \phi \mid \left[\textstyle\int e\tau k\right] \phi \mid \left\langle \textstyle\int e\tau k \right\rangle_< \phi \mid \left[\textstyle\int e\tau k\right]_< \phi \mid \\ \left\langle \textstyle\int e\tau k \right\rangle_> \phi \mid \left[\textstyle\int e\tau k\right]_> \phi \mid \neg\phi \mid \phi \wedge \psi \mid \phi \vee \psi\end{aligned}$$

The connectives $\rightarrow$ and $\leftrightarrow$ can be defined in usual way. For simplicity, we will denote $\left\langle \int e \geq 0 \right\rangle$ and $\left[\int e \geq 0\right]$ as $\langle e \rangle$ and $[e]$, respectively.

Before giving the formal semantics of EIL formulas, we will define an EIL model.

DEFINITION 3. Let $\mathcal{I}$ be the set of all bounded, closed and non-empty intervals of real numbers, and $\mathcal{E}$ be a finite set of event atoms. An EIL **model** $\mathcal{M}$ is a finite subset of $\mathcal{I} \times \mathcal{E}$.

As can be seen from the construction an EIL model, intervals are *primitive* objects of the model.

DEFINITION 4. Let $\mathcal{M}$ be an EIL model, $|J|$ denote the length of the interval J, and $I \in \mathcal{I}$. The formal semantics of EIL formulas is then defined as follows:

- $\mathcal{M} \models_I \left\langle \int e\tau k \right\rangle \phi$ iff $\exists J \subseteq I$ such that $\langle J, e\rangle \in \mathcal{M}$ and $|J|\,\tau k$ and $\mathcal{M} \models_J \phi$;
- $\mathcal{M} \models_I \left[\int e\tau k\right] \phi$ iff $\forall J \subseteq I$ $\langle J, e\rangle \in \mathcal{M}$ and $|J|\,\tau k$ imply $\mathcal{M} \models_J \phi$;
- $\mathcal{M} \models_I \left\langle \int e\tau k \right\rangle_< \phi$ iff $\exists J \subseteq I$ such that $\langle J, e\rangle \in \mathcal{M}$ and $|J|\,\tau k$ and $\mathcal{M} \models_{fin(J,I)} \phi$;
- $\mathcal{M} \models_I \left[\int e\tau k\right]_< \phi$ iff $\forall J \subseteq I$ $\langle J, e\rangle \in \mathcal{M}$ and $|J|\,\tau k$ imply $\mathcal{M} \models_{fin(J,I)} \phi$;
- $\mathcal{M} \models_I \left\langle \int e\tau k \right\rangle_> \phi$ iff $\exists J \subseteq I$ such that $\langle J, e\rangle \in \mathcal{M}$ and $|J|\,\tau k$ and $\mathcal{M} \models_{init(J,I)} \phi$;
- $\mathcal{M} \models_I \left[\int e\tau k\right]_> \phi$ iff $\forall J \subseteq I$ $\langle J, e\rangle \in \mathcal{M}$ and $|J|\,\tau k$ imply $\mathcal{M} \models_{init(J,I)} \phi$;
- $\mathcal{M} \models_I \neg\phi$ iff not $\mathcal{M} \models_I \phi$;
- $\mathcal{M} \models_I \phi \wedge \psi$ iff $\mathcal{M} \models_I \phi$ and $\mathcal{M} \models_I \psi$;
- $\mathcal{M} \models_I \phi \vee \psi$ iff $\mathcal{M} \models_I \phi$ or $\mathcal{M} \models_I \psi$.

One important characteristic of EIL formulas is the 'quasi-guarded' nature of the quantification they feature. Thus, for example, the formula $\langle e \rangle \phi$ existentially quantifies over intervals satisfying the event e (similarly for universal formulas). So it does not quantify over all subintervals of the current interval of evaluation without restriction. However, many modal logics, such as HS [5] and CDT [12], lack the 'quasi-guarded' character of the quantification that EIL formulas feature. This feature is very important to guarantee the decidability.

Before ending this section we will show how EIL represents English sentences including the temporal constructions. Consider the following sentences (in a fragment of English):

(2.1) `A warning is received during every control period until the water level becomes normal`.

(2.2) `After a drop in the water level, a warning is received during every control period until the water level becomes normal`.

The meaning of (2.1) is that, within the given temporal context I, there is an interval J over which the water level is normal; over every interval J', which is subsumed by the initial segment of I up to the beginning of I, a control period occurs; and J' subsumes some interval over which a warning is received. The sentence (2.1) is translated into EIL as follows:

$$\langle normal \rangle_< [control] \langle warning \rangle \top. \tag{2.3}$$

The sentence (2.2) can be represented by the following EIL formula:

$$\langle drop \rangle_> \langle normal \rangle_< [control] \langle warning \rangle \top. \tag{2.4}$$

Let's look at how EIL represents the event-based English sentences including duration. Consider the following sentence:

(2.5) `John solved a problem in less than ten minutes during every lunch break.`

This sentence can be translated into EIL as follows:

$$[break] \left\langle \textstyle\int solve \leq 10 \right\rangle \tag{2.6}$$

3 Finding a depth limit for EIL models

In this section we show that the depth of an EIL model is exponentially bounded by the length of a given formula φ whose satisfiability is checked.

We remark that the condition in Definition 3 that models are finite subsets of $\mathcal{I} \times \mathcal{E}$ is significant. Because there might be some EIL formulas which cannot be satisfied in a finite model. Consider, for example, the $\langle e \rangle \top \wedge [e] \langle e \rangle \top$. This formula is not satisfiable in a finite model; because it implies that every occurrence of e over an interval J requires another e to occur over a subinterval of J. Therefore, the formula is unsatisfiable in a finite model.

Below we will show how to normalize an EIL formula to the desired form.

LEMMA 5. *Every EIL formula is logically equivalent to one in which* $\neg$ *appears only in subformula of the form* $\bot$ $(= \neg\top)$.

Proof. The proof is trivial for $\bot$. In an EIL formula $\neg$ can be moved inwards as follows:

$\neg \left\langle \int e\tau k \right\rangle \phi \equiv \left[\int e\tau k \right] \neg\phi$; $\neg \left[\int e\tau k \right] \phi \equiv \left\langle \int e\tau k \right\rangle \neg\phi$;
$\neg \left\langle \int e\tau k \right\rangle_{<} \phi \equiv \left[\int e\tau k \right]_{<} \neg\phi$; $\neg \left[\int e\tau k \right]_{<} \phi \equiv \left\langle \int e\tau k \right\rangle_{<} \neg\phi$;
$\neg \left\langle \int e\tau k \right\rangle_{>} \phi \equiv \left[\int e\tau k \right]_{>} \neg\phi$; $\neg \left[\int e\tau k \right]_{>} \phi \equiv \left\langle \int e\tau k \right\rangle_{>} \neg\phi$. ∎

By means of Lemma we can normalize the forms of EIL formulas.

DEFINITION 6. Given an EIL formula φ and a non-empty model $\mathcal{M}$, the **depth** of $\mathcal{M}$ is the greatest m for which there exist $J_1 \subseteq ... \subseteq J_m$ such that for all i, $1 \leq i \leq m$ and for some $e \in \mathcal{E}$, $\langle J_i, e \rangle \in \mathcal{M}$. The depth of an empty model is defined to be 0.

LEMMA 7. *Let* φ *be an EIL formula.* φ *can be satisfied in a model* $\mathcal{M}$ *which is exponentially bounded by the length of* φ.

Proof. Assume φ has the form guaranteed by Lemma 5. Let m be the number of existential subformulas of φ ($\left\langle \int e\tau k \right\rangle \phi$, $\left\langle \int e\tau k \right\rangle_{<} \phi$ and $\left\langle \int e\tau k \right\rangle_{>} \phi$), and n be the number of universal subformulas of φ ($\left[\int e\tau k \right] \psi$, $\left[\int e\tau k \right]_{<} \psi$ and $\left[\int e\tau k \right]_{>} \psi$). An existential subformula $\left\langle \int e\tau k \right\rangle \phi$, similarly $\left\langle \int e\tau k \right\rangle_{<} \phi$ and $\left\langle \int e\tau k \right\rangle_{>} \phi$, implies that $\mathcal{M}$ contains an entry $\langle J, e \rangle$ for some interval J. Since φ is satisfiable, by semantics ϕ must be true at J. From this we can conclude that J subsumes a chain of intervals which satisfy event atoms occuring in ϕ. The length of such a chain is thus bounded by $|\varphi|$. So, every existential subformula of φ implies a chain of intervals, whose length is bounded by $|\varphi|$. In the worst case, these chains are aligned one under the other, and construct a longer chain, which is bounded by $m\,|\varphi|$.

Moreover, a universal subformula of the type $\left[\int e\tau k \right] \psi$ implies that ψ is true at each interval J satisfying e. We can therefore conclude that each J subsumes a chain of intervals which satisfy event atoms occuring in ψ ($\left[\int e\tau k \right]_{<} \psi$ and $\left[\int e\tau k \right]_{>} \psi$ can be considered similarly). In the worst case, the bound on the length of the whole chain increases to $m\,|\varphi|^2$. If we repeat the same step for the remaining $n-1$ universal subformulas we will see that the bound becomes k^n, where $k = m\,|\varphi|$. Since $m < |\varphi|$ and $n < |\varphi|$, we can conclude that $k^n < |\varphi|^{2|\varphi|}$. It easily follows that the depth bound of the model $\mathcal{M}$ is $2^{p(|\varphi|)}$, where p is a fixed polynomial. ∎

We remark that the exponential depth bound that we have found is not optimal. Here we have based our calculations into the worst case to find an upper limit, even if this case may be never encountered.

In the next section we will show that the size of this model is doubly exponential by the length of φ. We will actually derive the model from the tableau generated by a tableau procedure.

4 A tableau system procedure for EIL

In this section we propose a terminating tableau system for the logic EIL, thus showing that its satisfiability problem is decidable. Indeed, the satisfiability problem for EIL is in 2-NEXPTIME. This is proved by building models whose sizes are exponentially bounded.

In the following, we define a tableau-based decision procedure for EIL, and analyze its computational complexity. Then, we prove its soundness and completeness. The procedure is based on an *expansion strategy.* The expansion strategy involves three rules: the *interval relation rule,* which nondeterministically guesses the interval relation among nodes in the graph, the *existential node expansion rule*, which expands existential subformulas in a node and the *universal node expansion rule*, which expands universal subformulas in a node. A *blocking* condition guarantees the termination of the method.

4.1 Preliminary notions

In the following we introduce some preliminary notions which will be used throughout the rest of the paper.

DEFINITION 8.

A **successor** of a node v is a node w such that there is an edge from v to w. A **path** is a sequence of nodes $v_1, ..., v_k$ such that for all $1 \leq i < k$, v_{i+1} is a successor of v_i. The **depth** of a node v is the maximum number of edges of a path from the root node to v.

DEFINITION 9.

A **decorated graph** $\mathcal{G}$ is a graph in which every node has a decoration. For a node $v \in \mathcal{G}$, a **decoration** $\lambda(v)$ is a 5-tuple $([b_v, e_v], \rho(v), \mathcal{K}(v), \mathcal{L}(v), \mathcal{L}'(v))$, where b_v (e_v) is a *constraint variable* denoting the beginning (ending) of the interval represented by the node v, $\rho(v)$ denotes the label of the node v (where $\rho(v) \in \mathcal{E}$), $\mathcal{K}(v)$ denotes a formula associated with the node v, and $\mathcal{L}(v)$ and $\mathcal{L}'(v)$ denote a set of subformulas associated with the node v.

DEFINITION 10.

A **temporal constraint** is a relation involving constraint variables which denote interval endpoints.

For example, the temporal constraint $b_v \geq b_u, e_v \leq e_u$ shows an interval relation between $[b_v, e_v]$ and $[b_u, e_u]$.

DEFINITION 11.

A **tableau** for a given formula φ is a tuple $\langle \mathcal{G}, \mathcal{C} \rangle$, where $\mathcal{G}$ denotes a decorated graph, and $\mathcal{C}$ denotes the set of temporal constraints in the graph $\mathcal{G}$.

4.2 Tableau method

Let φ be a formula to be checked for satisfiability over an interval I_0. The *initial tableau* for φ is the tuple $\langle v_0, \mathcal{C}_0 \rangle$, where v_0 is the initial graph with the decoration $\lambda(v_0) = ([b_{v_0}, e_{v_0}], \rho(v_0), \mathcal{K}(v_0), \mathcal{L}(v_0), \mathcal{L}'(v_0))$ such that $\rho(v_0) = root$, $\mathcal{K}(v_0) = \varphi$, $\mathcal{L}(v_0) = \emptyset$, $\mathcal{L}'(v_0) = \emptyset$, and $\mathcal{C}_0$ is the initial set of temporal constraints such that $\mathcal{C}_0 = \{b_{v_0} = start(I_0), e_{v_0} = end(I_0)\}$. Assume Q denotes the queue of nodes in $\mathcal{G}$ awaiting processing. Then, the inital value of Q is $\{v_0\}$.

A tableau for φ is a tuple $\langle \mathcal{G}, \mathcal{C} \rangle$, where $\mathcal{C}$ is obtained by expanding the initial constraint set $\mathcal{C}_0$ with temporal constraints in the existing nodes, and the decorated graph $\mathcal{G}$ is obtained by expanding the initial node v_0 through successive applications of the *expansion strategy* to existing nodes until no node remains to process. In other words, the expansion strategy is applied to every node in Q until $Q = \emptyset$. When a node is selected, it is removed from Q.

During the application of the expansion strategy to a node, we need to solve the temporal constraints in $\mathcal{C}$. Remember that each node of the graph represents an interval. For our purposes, we model intervals as pairs of endpoints, which are distinct numbers on the real line. Let $T = \{b_{v_1}, ..., b_{v_n}, e_{v_1}, ..., e_{v_n}\}$ be a set of constraint variables. The constraints of a tableau can be represented as a Simple Temporal Problem [3]. Given that n is the number of variables the complexity of a solution to a STP (if there is any) can be found in $\mathcal{O}(n^3)$ time and $\mathcal{O}(n^2)$ space. If the set of temporal constraints in $\mathcal{C}$ is inconsistent, then a solution will not be found, and we say $\mathcal{C}$ is not satisfiable.

In order to avoid infinite paths, and therefore to have a finite satisfying model we need to guarantee the termination of the proposed tableau method below. In the following we give a suitable *stopping condition* for the tableau procedure:

DEFINITION 12. A tableau $\langle \mathcal{G}, \mathcal{C} \rangle$ is **closed** if one of the following conditions hold:

- $\perp \in \mathcal{L}(v)$ for some node v in $\mathcal{G}$,
- $\mathcal{C}$ is not satisfiable,
- The depth of the shortest path $v_0 \rightarrow ... \rightarrow v$ is more than $|\varphi|^2$ for some node v in $\mathcal{G}$ (where v_0 is the root node.)

DEFINITION 13. A tableau is **open** if it is not closed.

Once the tableau procedure terminates, we check whether the tableau generated is open. For a given formula φ if there is an open tableau, then φ is satisfiable, and the satisfying model $\mathcal{M}$ is derived from the tableau. We do this by picking some solution σ, which assigns real values to constraint variables in $\mathcal{C}$. Let $J_v = [\sigma(b_v), \sigma(e_v)]$ be the interval represented by a node v of $\mathcal{G}$. We construct a model $\mathcal{M}$ as follows: $\mathcal{M} = \{\langle J_v, \rho(v) \rangle \mid \text{for any } v \in \mathcal{G}$ s.t. $\rho(v) \notin \{root, -\}\}$. If the tableau is closed, then φ is unsatisfiable.

Expansion strategy.

Let $\langle \mathcal{G},\mathcal{C}\rangle$ be a tableau, v be a node in $\mathcal{G}$ with $\lambda(v) = ([b_v, e_v], \rho(v), \mathcal{K}(v), \mathcal{L}(v), \mathcal{L}'(v))$, and Q be the queue of nodes awaiting processing. We say the **expansion strategy** for a node v is defined as follows:

If the tableau is open, apply the following rules:

Rule 1. Set $Q := Q \setminus \{v\}$, and apply the **interval relation rule** to the node v.

Rule 2. Let the Disjunctive Normal Form (DNF) of $\mathcal{K}(v)$ be $\psi_1 \vee ... \vee \psi_n$ where $\psi_i = \psi_{i1} \wedge ... \wedge \psi_{in_i}$ ($n \geq 1$,$1 \leq i \leq n$ and $n_i \geq 1$). Select some i, and set $\mathcal{L}'(v) := \{\psi_{i1}, ..., \psi_{in_i}\}$, $\mathcal{L}(v) := \mathcal{L}(v) \cup \mathcal{L}'(v)$ and $\mathcal{K}(v) := \top$.

Rule 3. Apply the **universal node expansion rule** to the node v.

Rule 4. Apply the **existential node expansion rule** to the node v.

Interval relation rule.

The *interval relation rule* guesses the interval relation between the given node and all other nodes in the graph. In [1] Allen introduced well-known thirteen different binary relations between intervals on a linear ordering, which are *before*, *meets*, *overlaps*, *starts*, *during*, *finishes*, *equals*, *finished by*, *during by*, *started by*, *overlapped by*, *met by* and *after*.

Let $\langle \mathcal{G},\mathcal{C}\rangle$ be a tableau, and v be a node in $\mathcal{G}$ with $\lambda(v) = ([b_v, e_v], \rho(v), \mathcal{K}(v), \mathcal{L}(v), \mathcal{L}'(v))$. Assume τ' is the corresponding inverted operator of τ (where $\tau \in \{<, \leq, =, \geq, >\}$). The *interval relation rule* for a node v is defined as follows:

For any node u (except v) in $\mathcal{G}$

If there is no edge from u to v, or from v to u, then nondeterministically guess the interval relation between u and v:

v *before* u : Set $\mathcal{C} := \mathcal{C} \cup \{e_v < b_u\}$.

v *meets* u : Set $\mathcal{C} := \mathcal{C} \cup \{e_v = b_u\}$.

v *non-strict-during* u : Set $\mathcal{C} := \mathcal{C} \cup \{b_v \geq b_u, e_v \leq e_u\}$, and add an edge from u to v ($u \rightarrow v$).

- **if** $\rho(v) = e$ and $\left[\int e\tau k\right] \psi \in \mathcal{L}(u)$, **then** set either $i)$ $\mathcal{C} := \mathcal{C} \cup \{(e_v - b_v)\tau' k\}$; or $ii)\mathcal{C} := \mathcal{C} \cup \{(e_v - b_v)\tau k\}$ and $\mathcal{K}(v) := \mathcal{K}(v) \wedge \psi$.

- **if** $\rho(v) = e$ and $\left[\int e\tau k\right]_< \psi \in \mathcal{L}(u)$, **then** for every $\left[\int e\tau k\right]_< \psi \in \mathcal{L}(u)$ do either $i)$ set $\mathcal{C} := \mathcal{C} \cup \{(e_v - b_v)\tau' k\}$; or $ii)$ set $\mathcal{C} := \mathcal{C} \cup \{(e_v - b_v)\tau k\}$, add an immediate successor w with $\rho(w) = -$, $\mathcal{K}(w) = \psi$, $\mathcal{L}(w) = \emptyset$, $\mathcal{L}'(w) = \emptyset$, set $\mathcal{C} := \mathcal{C} \cup \{b_w = e_v, e_w = e_u\}$, add an edge from u to w and v to w ($u \rightarrow w$,$v \rightarrow w$), and set $Q := Q \cup \{w\}$.

- **if** $\rho(v) = e$ and $\left[\int e\tau k\right]_> \psi \in \mathcal{L}(u)$, **then** follow the step above; except that rather than setting $\mathcal{C} := \mathcal{C} \cup \{b_w = e_v, e_w = e_u\}$, set $\mathcal{C} := \mathcal{C} \cup \{b_w = b_v, e_w = b_u\}$.

v *overlaps* u : Set $\mathcal{C} := \mathcal{C} \cup \{b_v < b_u < e_v < e_u\}$, and add an edge from u to v ($u \rightarrow v$).

Given two intervals J_1 and J_2, we say J_1 *non-strict-during* J_2 if J_1 is a non-strict subinterval of J_2. Once we guess the interval relation as "non-strict-during", we do not need to consider the relations "equals", "during", "starts", "started-by", "finishes" and "finished-by". The cases where v "after" u, v "met-by" u, v "includes" u and v "overlapped-by" u can be dealt with similarly. Note that in the interval relation rule, we consider the possibility that $\mathcal{L}(u)$ of an existing node u includes a universal subformula which might update the decoration of the node v.

Please note that when we denote an interval relation between two nodes, such as v "during" u, we mean this interval relation holds between the intervals represented by these nodes. For simplicty, we will use this adaption.

Universal node expansion rule.

The *universal node expansion rule* expands all universal subformulas in $\mathcal{L}'(v)$. Let $\langle \mathcal{G}, \mathcal{C} \rangle$ be a tableau, and v be a node in $\mathcal{G}$ with $\lambda(v) = ([b_v, e_v], \rho(v), \mathcal{K}(v), \mathcal{L}(v), \mathcal{L}'(v))$. Assume τ' is the corresponding inverted operator of τ (where $\tau \in \{<, \leq, =, \geq, >\}$). The *universal node expansion rule* for a node v is defined as follows:

For every $\xi \in \mathcal{L}'(v)$

if $\xi = \left[\int e\tau k\right] \psi$, **then for every** node u (except v) in $\mathcal{G}$ with $\rho(u) = e$ and u *non-strict-during* v, set either *i)* $\mathcal{C} := \mathcal{C} \cup \{(e_u - b_u)\tau' k\}$; or *ii)* $\mathcal{C} := \mathcal{C} \cup \{(e_u - b_u)\tau k\}$, $\mathcal{K}(u) := \mathcal{K}(u) \wedge \psi$ and $Q := Q \cup \{u\}$.

if $\xi = \left[\int e\tau k\right]_< \psi$, **then for every** node u (except v) in $\mathcal{G}$ with $\rho(u) = e$ and u *non-strict-during* v, do either *i)* set $\mathcal{C} := \mathcal{C} \cup \{(e_u - b_u)\tau' k\}$; or *ii)* set $\mathcal{C} := \mathcal{C} \cup \{(e_u - b_u)\tau k\}$, add an immediate successor w with $\rho(w) = -$, $\mathcal{K}(w) = \psi$, $\mathcal{L}(w) = \emptyset$, $\mathcal{L}'(w) = \emptyset$, set $\mathcal{C} := \mathcal{C} \cup \{b_w = e_u, e_w = e_v\}$ and set $Q := Q \cup \{w\}$.

where u "non-strict-during" v is true if $b_w \geq b_v, e_w \leq e_v \in \mathcal{C}$. The case where $\xi = \left[\int e\tau k\right]_> \psi$ can be dealt with similarly. As a result of applying the universal node expansion rule, some of the existing nodes might be revisited, which means we re-execute the expansion strategy for these nodes. In this case, interval relations will not be guessed again; but their decoration might get updated.

Existential node expansion rule.

The *existential node expansion rule* expands all existential subformulas in $\mathcal{L}'(v)$. Let $\langle \mathcal{G}, \mathcal{C} \rangle$ be a tableau, and v be a node in $\mathcal{G}$ with $\lambda(v) = ([b_v, e_v], \rho(v), \mathcal{K}(v), \mathcal{L}(v), \mathcal{L}'(v))$. Assume τ' is the corresponding inverted operator of τ (where $\tau \in \{<, \leq, =, \geq, >\}$). The *existential node expansion rule* for a node v is defined as follows:

For every $\xi \in \mathcal{L}'(v)$

if $\xi = \left\langle \int e\tau k \right\rangle \psi$, **then** add an immediate successor w with $\rho(w) = e$, $\mathcal{K}(w) = \psi$, $\mathcal{L}(w) = \emptyset$, $\mathcal{L}'(w) = \emptyset$, set $\mathcal{C} := \mathcal{C} \cup \{b_w \geq b_v, e_w \leq e_v, (e_w - b_w)\tau k\}$, and set $Q := Q \cup \{w\}$.

if $\xi = \left\langle \int e\tau k \right\rangle_< \psi$, **then** add two immediate successors w, w' with $\rho(w) = e$, $\mathcal{K}(w) = \emptyset$, $\mathcal{L}(w) = \emptyset$, $\mathcal{L}'(w) = \emptyset$ and $\rho(w') = -$, $\mathcal{K}(w') = \psi$, $\mathcal{L}(w') = \emptyset$,

$\mathcal{L}'(w') = \emptyset$, set $\mathcal{C} := \mathcal{C} \cup \{b_w \geq b_v, e_w \leq e_v, b_{w'} = e_w, e_{w'} = e_v\}$, and set $Q := Q \cup \{w, w'\}$.

The case where $\xi = \left\langle \int e\tau k \right\rangle_{>} \psi$ can be dealt with similarly. The existential node expansion rule creates a new node (or nodes). In the next run, we apply the expansion strategy to this node, and the decoration of this node gets updated according to Rule 2.

4.3 Soundness and completeness

The soundness and completeness of the proposed tableau method is proved below. But we first prove the termination of the method.

THEOREM 14. *The tableau method for EIL terminates.*

Proof. Let $\langle \mathcal{G}, \mathcal{C} \rangle$ be a tableau constructed by the tableau procedure for a given a formula φ. By the stopping condition in the tableau procedure every node of $\mathcal{G}$ has a finite outgoing degree and every branch of it is of finite length. Therefore, the tableau method terminates. ■

THEOREM 15. *Let φ be an EIL formula which has the form guaranteed by Lemma 5. φ is satisfiable iff there is an open tableau for φ.*

Proof. Soundness $(\Leftarrow)$ **:**

Suppose $\langle \mathcal{G}, \mathcal{C} \rangle$ is an open tableau for φ. We pick some solution $\sigma : \mathcal{V} \to \mathbb{R}$, which assigns real values to constraint variables in $\mathcal{C}$. Let $J_v = [\sigma(b_v), \sigma(e_v)]$ be the interval represented by the node v of $\mathcal{G}$. We construct a model $\mathcal{M}$ as follows: $\mathcal{M} = \{\langle J_v, \rho(v) \rangle \mid$ for any $v \in \mathcal{G}$ s.t. $\rho(v) \notin \{root, -\}\}$.

Now we show that $\mathcal{M} \models_{I_0} \varphi$ (where I_0 is the initial interval). We claim that for every v in $\mathcal{G}$, $\mathcal{M} \models_{J_v} \mathcal{L}(v)$. We show, by structural induction, that $\phi \in \mathcal{L}(v)$ implies $\mathcal{M} \models_{J_v} \phi$. Note that, by construction of the tableau, $\mathcal{L}(v)$ comprises the formulas are of the forms $\top$, $\bot$, $\left\langle \int e\tau k \right\rangle \psi$, $\left\langle \int e\tau k \right\rangle_{<} \psi$, $\left\langle \int e\tau k \right\rangle_{>} \psi$, $\left[\int e\tau k \right] \psi$, $\left[\int e\tau k \right]_{<} \psi$ and $\left[\int e\tau k \right]_{>} \psi$.

Base Case:

$\phi = \top$: Trivial

$\phi = \bot$: Since $\langle \mathcal{G}, \mathcal{C} \rangle$ is an open tableau, by definition 12 and 13, $\bot \notin \mathcal{L}(v)$.

Inductive Case:

$\phi = \left\langle \int e\tau k \right\rangle \psi$: By the existential node expansion rule, there exists a node w with $\rho(w) = e$ and $\mathcal{K}(w) = \psi$. In addition, $\mathcal{C}$ contains $b_w \geq b_v, e_w \leq e_v$ and $(e_w - b_w)\tau k$. Let ψ be $\psi_1 \vee ... \vee \psi_n$ where $\psi_i = \psi_{i1} \wedge ... \wedge \psi_{in_i}$ ($n \geq 1$,$1 \leq i \leq n$ and $n_i \geq 1$). By Rule 2, $\psi_{i1}, .., \psi_{in_i} \in \mathcal{L}(w)$ for some i ($1 \leq i \leq n$). By the inductive hypothesis, $\mathcal{M} \models_{J_w} \psi_{i1} \wedge ... \wedge \mathcal{M} \models_{J_w} \psi_{in_i}$. Therefore, $\mathcal{M} \models_{J_w} \psi$. By construction, we have $\langle J_w, e \rangle \in \mathcal{M}$ with $|J_w| \tau k$ and $J_w \subseteq J_v$. Thus, $\mathcal{M} \models_{J_v} \phi$.

$\phi = \left\langle \int e\tau k \right\rangle_{<} \psi$ and $\phi = \left\langle \int e\tau k \right\rangle_{>} \psi$: Similar to the case $\phi = \left\langle \int e\tau k \right\rangle \psi$.

$\phi = \left[\int e\tau k \right] \psi$: By the construction of $\mathcal{M}$, for any $J \in \mathcal{I}$ if $\langle J, e \rangle \in \mathcal{M}$, then there exists a node u in $\mathcal{G}$ such that $J_u = J$. According to the universal node expansion rule (or the interval relation rule) if $J_u \subseteq J_v$, then we do either: *i*) set $\mathcal{C} := \mathcal{C} \cup \{(e_u - b_u)\tau' k\}$ (τ' is the corresponding inverted operator of τ); or *ii*) set $\mathcal{C} := \mathcal{C} \cup \{(e_u - b_u)\tau k\}$ and $\mathcal{K}(u) := \mathcal{K}(u) \wedge \psi$.

Assume $|J_u|\,\tau k$ is false. Whatever the choice is, it is trivial to see that $\langle J_u, e\rangle \in \mathcal{M}$, $J_u \subseteq J_v$ and $|J_u|\,\tau k$ imply $\mathcal{M} \models_{J_u} \psi$. Assume $|J_u|\,\tau k$ is true. In this case, option *i* mentioned above cannot have been selected. Otherwise, $\mathcal{C}$ would contain $\{(e_u - b_u)\tau' k\}$, and it would result in an inconsistency. So option *ii* has been taken. In this case, we set $\mathcal{C} := \mathcal{C} \cup \{(e_u - b_u)\tau k\}$ and $\mathcal{K}(u) := \mathcal{K}(u) \wedge \psi$. Let ψ be $\psi_1 \vee ... \vee \psi_n$ where $\psi_i = \psi_{i1} \wedge ... \wedge \psi_{in_i}$ ($n \geq 1$,$1 \leq i \leq n$ and $n_i \geq 1$). By Rule 2, $\psi_{i1}, .., \psi_{in_i} \in \mathcal{L}(u)$ for some i ($1 \leq i \leq n$). By the inductive hypothesis, $\mathcal{M} \models_{J_u} \psi$. By construction, we have $\langle J_u, e\rangle \in \mathcal{M}$. We also know that $J_u \subseteq J_v$ and $|J_u|\,\tau k$. Therefore, for any witness J_u, $\langle J_u, e\rangle \in \mathcal{M}$, $J_u \subseteq J_v$ and $|J_u|\,\tau k$ imply $\mathcal{M} \models_{J_u} \psi$. Thus, $\mathcal{M} \models_{J_u} \phi$.

$\phi = \left[\int e\tau k\right]_< \psi$ and $\phi = \left[\int e\tau k\right]_> \psi$: Similar to the case $\phi = \left[\int e\tau k\right] \psi$.

We have proved that for every v in $\mathcal{G}$, $\mathcal{M} \models_{J_v} \mathcal{L}(v)$. In particular, $\mathcal{M} \models_{I_0} \mathcal{L}(v_0)$. We know that $\mathcal{K}(v_0) = \varphi$. Now assume $\varphi = \varphi_1 \vee ... \vee \varphi_n$, where $\varphi_i = \varphi_{i1} \wedge ... \wedge \varphi_{in_i}$ ($n \geq 1$, $1 \leq i \leq n$ and $n_i \geq 1$). According to Rule 2, $\mathcal{L}(v_0) = \{\varphi_{i1}, ..., \varphi_{in_i}\}$ for some value of i. Therefore, we can easily conclude that $\mathcal{M} \models_{I_0} \varphi$.

Completeness ($\Rightarrow$) **:**

Let φ be a satisfiable formula, and I_0 be an interval. By Lemma 7 φ can be satisfied by a model $\mathcal{M}$, which has a depth bound of $2^{p(|\varphi|)}$ for a fixed polynomial p, such that $\mathcal{M} \models_{I_0} \varphi$. We will show that there is an open tableau $\langle \mathcal{G}, \mathcal{C}\rangle$ for φ.

The *initial tableau* for φ is the tuple $\langle v_0, \mathcal{C}_0\rangle$, where v_0 is the initial graph such that $\mathcal{K}(v_0) = \varphi$ and $\mathcal{L}(v_0) = \emptyset$, and $\mathcal{C}_0$ is the initial set of temporal constraints such that $\mathcal{C}_0 = \{b_{v_0} = \mathit{start}(I_0), e_{v_0} = \mathit{end}(I_0)\}$. According to the expansion strategy we apply the interval relation rule to the node v_0 as $\mathcal{L}(v_0)$ is empty. But since there is only one node, $\mathcal{K}(v_0)$ does not get updated. Let the disjunctive normal form of $\mathcal{K}(v_0) = \varphi$ be $\varphi_1 \vee ... \vee \varphi_n$, where $\varphi_i = \varphi_{i1} \wedge ... \wedge \varphi_{in_i}$ ($n \geq 1$, $1 \leq i \leq n$ and $n_i \geq 1$). Since $\mathcal{M} \models_{I_0} \varphi$, $\mathcal{M} \models_{I_0} \varphi_i$ for at least one value of i. So in Rule 2 we pick this value of i, so that $\mathcal{L}(v_0) = \{\varphi_{i1}, ..., \varphi_{in_i}\}$.

Now, we claim that for each node v in $\mathcal{G}$, there exists an interval J_v such that $\mathcal{M} \models_{J_v} \mathcal{L}(v)$ (Once we pick a witness J_v, it remains assigned to the node v until the tableau procedure terminates.) We prove the claim by induction on the stage in tableau construction at which the node v was created.

Base case:

Above we have shown that $\mathcal{M} \models_{I_0} \varphi_i$ for some value of i, and $\mathcal{L}(v_0) = \{\varphi_{i1}, ..., \varphi_{in_i}\}$. So, it is trivial to see $\mathcal{M} \models_{I_0} \mathcal{L}(v_0)$.

Inductive case:

Case 1: Let w be a node in $\mathcal{G}$ such that $\rho(w) = e$. Then w must have been created by the existential node expansion rule applied to a node v of which w is a successor node. After the node w has been created, we apply the expansion strategy to the node w. So we first apply the interval relation rule. Let us consider two cases:

i) Application of the interval relation rule adds no material to $\mathcal{L}(w)$: Assume $\mathcal{L}(w) = \{\psi_0\}$ where $\psi_0 = \psi_{01} \wedge ... \wedge \psi_{0n_0}$ $(n_0 \geq 1)$. In this case, $\mathcal{L}(v)$ must contain $\xi = \left\langle \int e\tau k \right\rangle \psi$ where ψ has the form $\psi_0 \vee ... \vee \psi_l$ $(l \geq 0)$ (If $\mathcal{L}(v)$ contained $\left\langle \int e\tau k \right\rangle_< \psi$ or $\left\langle \int e\tau k \right\rangle_> \psi$, then the existential rule would set $\rho(w) = -$. But we already know that $\rho(w) = -$. So, $\mathcal{L}(v)$ can contain neither $\left\langle \int e\tau k \right\rangle_< \psi$ nor $\left\langle \int e\tau k \right\rangle_> \psi$). By the inductive hypothesis a witness J_v is defined such that $\mathcal{M} \models_{J_v} \mathcal{L}(v)$. Let J_w be an interval for the node w. Thus, $\mathcal{M} \models_{J_w} \psi$.

When the existential rule was applied to v, we set $\mathcal{K}(w) := \psi$ and $\mathcal{C} := \mathcal{C} \cup \{b_w \geq b_v, e_w \leq e_v, (e_w - b_w)\tau k\}$. According to Rule 2 we select some of the disjunct of ψ, and extend $\mathcal{L}(w)$ with this disjunct. It is clear that ψ_0 is the subformula which was selected. So, $\mathcal{M} \models_{J_w} \psi_0$. Hence, $\mathcal{M} \models_{J_w} \mathcal{L}(w)$.

ii) Application of the interval relation rule adds some material to $\mathcal{L}(w)$: Assume $\mathcal{L}(w) = \{\psi_0, \psi_1, ..., \psi_m\}$ where $\psi_i = \psi_{i1} \wedge ... \wedge \psi_{im_i}$ $(0 \leq i \leq m$ and $m_i \geq 1)$, ψ_0 has been added to $\mathcal{L}(w)$ by applying the existential rule in v, and $\psi_1, ..., \psi_m$ have been added to $\mathcal{L}(w)$ by applying the interval relation rule to the node w. Above we have shown that $\mathcal{M} \models_{J_w} \psi_0$.

According to the interval relation rule we guess the interval relation between w and any node in $\mathcal{G}$. Assume for any $1 \leq j \leq m$ ψ_j has been added to $\mathcal{L}(w)$ as a result of guessing the interval relation between w and a node u_j. Since $\mathcal{K}(w)$, and therefore $\mathcal{L}(w)$, has been updated, this relation must have been "non-strict-during". In this case, $\mathcal{L}(u_j)$ must contain $\xi = \left[\int e\tau k\right] \psi$, where ψ has the form $\psi_j \vee ... \vee \psi_{j+l}$ $(l \geq 0)$. By the inductive hypothesis we have picked a witness J_{u_j} such that $\mathcal{M} \models_{J_{u_j}} \mathcal{L}(u_j)$; thus $\mathcal{M} \models_{J_{u_j}} \xi$. We know that $J_w \subseteq J_{u_j}$ because in the interval rule we have guessed the relation between J_w and J_{u_j} as "non-strict-during" (As we can see in the interval rule, $\mathcal{C}$ has been updated according to the corresponding non-deterministic choice of the relation.) We also know that $|J_w|\,\tau k$ because we have selected the option *ii* in the interval relation rule, and set $\mathcal{C} := \mathcal{C} \cup \{(e_w - b_w)\tau k\}$ (Otherwise, $\mathcal{K}(w)$ could not have been updated). Therefore, $\mathcal{M} \models_{J_w} \psi$.

When the interval rule was applied to w, we set $\mathcal{K}(w) := \mathcal{K}(w) \wedge \psi$. It is clear that ψ_j was selected when the Rule 2 the expansion strategy was applied. Thus, for any $1 \leq j \leq m$ $\mathcal{M} \models_{J_w} \psi_j$. Hence, $\mathcal{M} \models_{J_w} \mathcal{L}(w)$.

So, we have shown that once a node w is created, and the expansion strategy is applied, it is true that $\mathcal{M} \models_{J_w} \mathcal{L}(w)$. However, when new nodes are added to $\mathcal{G}$, $\mathcal{L}(w)$ might get updated through the application of the universal node expansion rule in these nodes. So, we must show that whenever new material is added to $\mathcal{L}(w)$, $\mathcal{M} \models_{J_w} \mathcal{L}(w)$ remains true.

Now, assume $\mathcal{L}(w) = \{\psi_0, ..., \psi_m, \psi_{m+1}, ..., \psi_{m+n}\}$ where $\psi_i = \psi_{i1} \wedge ... \wedge \psi_{in_i}$ $(0 \leq i \leq m+n$ and $n_i \geq 1)$, and $\psi_{m+1}, ..., \psi_{m+n}$ have been added to $\mathcal{L}(w)$ by applying the universal node expansion rule to some nodes in $\mathcal{G}$. Above we have shown that $\mathcal{M} \models_{J_w} \{\psi_0, ..., \psi_m\}$. Assume for any $m+1 \leq$

$k \leq m+n$, ψ_k has been added to $\mathcal{L}(w)$ by applying the universal node expansion rule to a node u_k in $\mathcal{G}$. In this case, $\mathcal{L}(u_k)$ must contain $\xi = \langle e \rangle \psi$, where ψ has the form $\psi_k \vee ... \vee \psi_{k+l}$ ($l \geq 0$). By the inductive hypothesis we have picked a witness J_{u_k} such that $\mathcal{M} \models_{J_{u_k}} \mathcal{L}(u_k)$; thus $\mathcal{M} \models_{J_{u_k}} \xi$. We know that $J_w \subseteq J_{u_k}$. We also know that $|J_w| \tau k$ because we have selected the option *ii* of the universal rule, and set $\mathcal{C} := \mathcal{C} \cup \{(e_w - b_w)\tau k\}$ (Otherwise, $\mathcal{K}(w)$ could not have been updated.) Therefore, $\mathcal{M} \models_{J_w} \psi$.

When the universal rule was applied to u_k, we set $\mathcal{K}(w) := \mathcal{K}(w) \wedge \psi$. It is clear that ψ_k was selected when Rule 2 of the expansion strategy was applied. So, for any $m+1 \leq k \leq m+n$ $\mathcal{M} \models_{J_w} \psi_k$. Hence, $\mathcal{M} \models_{J_w} \mathcal{L}(w)$.

Case 2: Let w be a node in $\mathcal{G}$ such that $\rho(w) = -$. Assume $\mathcal{L}(w) = \{\psi_0\}$ where $\psi_0 = \psi_{01} \wedge ... \wedge \psi_{0n_0}$ ($n_0 \geq 1$). Then, the dummy node w must have been created by either the existential node expansion rule, the interval relation rule, or the universal node expansion rule. If it has been created by the existential rule, then $\mathcal{L}(v)$ of a node v of which w is a successor node must contain either $\xi = \left\langle \int e\tau k \right\rangle_< \psi$ or $\xi = \left\langle \int e\tau k \right\rangle_> \psi$. Otherwise, $\mathcal{L}(u)$ of a node u at which the interval relation rule or the universal rule has been applied contains either $\xi = \left[\int e\tau k\right]_< \psi$ or $\xi = \left[\int e\tau k\right]_> \psi$. In each case, by the inductive hypothesis a witness J_v (J_u) is defined such that $\mathcal{M} \models_{J_v} \mathcal{L}(v)$ ($\mathcal{M} \models_{J_u} \mathcal{L}(u)$). By construction, there exists a node w' with $\rho(w') = e$. Let $J_{w'}$ be an interval for the w'. It is trivial to see that $J_{w'} \subseteq J_v$ ($J_{w'} \subseteq J_u$) and $|J_{w'}| \tau k$. Since $\mathcal{M} \models_{J_v} \xi$, $\mathcal{M} \models_{J_w} \psi$, where $J_w = \hbar(J_{w'}, J_v)$ or ($J_w = \hbar(J_{w'}, J_u)$). Here the partial function $\hbar$ is fin if $\xi = \left\langle \int e\tau k \right\rangle_< \psi$, and it is $init$, otherwise.

When any of these rules (existential rule, universal rule and interval rule) was applied, we set $\mathcal{K}(w) := \mathcal{K}(w) \wedge \psi$. Suppose ψ have the form $\psi_0 \vee ... \vee \psi_l$ ($l \geq 0$). Since ψ_0 is the selected disjunct of ψ in Rule 2, $\mathcal{M} \models_{J_w} \psi_0$. Hence, $\mathcal{M} \models_{J_w} \mathcal{L}(w)$.

Therefore, we have proved that for each node v in $\mathcal{G}$, there exists an interval J_v such that $\mathcal{M} \models_{J_v} \mathcal{L}(v)$.

Meanwhile, we know the depth of the model $\mathcal{M}$ is at most of order $2^{p(|\varphi|)}$ by the assumption. Since for any node v in $\mathcal{G}$, $\mathcal{M} \models_{J_v} \mathcal{L}(v)$, $\perp$ cannot be contained in $\mathcal{L}(v)$. As we have a witness J_v for each node v, we must have a solution for $\mathcal{C}$. Therefore, $\mathcal{C}$ must be satisfiable. Because none of the conditions in Definition 12 holds, it follows that $\langle \mathcal{G}, \mathcal{C} \rangle$ is an open tableau. ■

4.4 Computational complexity

THEOREM 16. *The satisfiability problem for EIL is in 2-NEXPTIME.*

Proof. In Theorem 14 we show that the proposed method terminates. Now, we analyse its computational complexity. We now give a bound on the size of any tableau for φ.

The out degree of any node is bounded by $|\varphi|$. The depth of the longest path in the tableau is bounded by $2^{p(|\varphi|)}$ for a fixed polynomial p by Lemma

7. Therefore, the size of the tableau is bounded by $|\varphi|^{2^{p(|\varphi|)}} = 2^{2^{p(|\varphi|)} log_2 |\varphi|}$. So, the tableau procedure builds a tableau of size $2^{2^{p'(|\varphi|)}}$ for some fixed polynomial p'. We can say that if an EIL formula φ is satisfiable, then the tableau procedure construct a graph, from which a satisfying model $\mathcal{M}$ is extracted, which has doubly exponential size by the length of φ. ■

5 Conclusion

In this paper we introduced an interval temporal logic EIL to represent meanings of sentences in English. EIL has affinity with the syntax of temporal constructions in English, and which is convenient for expressing the semantics of natural language constructions. EIL is interpreted over a linear time flow with only finitely many events able to occur over a bounded-time interval. EIL also employs the notion of duration.

In order to bound models we showed that the depth of an EIL model is exponentially bounded by the length of a given formula. We also proposed a terminating tableau system for the logic EIL, thus showing that its satisfiability problem is decidable. Indeed, it was proved that the satisfiability problem for EIL is in 2-NEXPTIME. This was proved by building models, which have doubly exponential size by the length of the given formula.

The future research directions include extending EIL with states and state models to specify real-time system requirements, finding a lower bound for the complexity of the satisfiability problem, introducing an axiomatization system to complement the semantic view, and comparing expressive powers of EIL with the related interval temporal logics.

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Savas Konur
Department of Computer Science
University of Manchester
Manchester, M13 9PL, UK
konurs@cs.man.ac.uk